Zipping Segment Trees

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Dynamic Segment Trees  [van Kreveld, Overmars, JACM 1993]

[1, 10)  [5, 8)  [6, 20)  [15, 25)
Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]

[1, 10)  [5, 8)  [6, 20)  [15, 25)

Stabbing Query

Given a set of intervals $\mathcal{M}$ and a point $p$, find all intervals $I \in \mathcal{M}$ with $p \in I$. 
Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]

\[ [1, 10) \quad [5, 8) \quad [6, 20) \quad [15, 25) \]

\[ 1 \quad 5 \quad 6 \quad 8 \quad 10 \quad 15 \quad 20 \quad 25 \]
Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]
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van Kreveld, Overmars, JACM 1993
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Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]
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"Weak Segment Tree Property"
Dynamic Segment Trees

van Kreveld, Overmars, JACM 1993
Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]

Balance!
Dynamic Segment Trees

van Kreveld & Overmars’ solution
- Use Red-Black Trees
- Repair annotations after rebalancing

Balance!
Zip Trees — Insertion

[Tarjan et al. WADS 2019.]
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Smaller than 2

Larger than 2
Zip Trees — Insertion

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Zip Trees — Insertion

 TARJAN ET AL. WADS 2019.

 Smaller than 2

 Larger than 2

 1 5

 3
Zipping Segment Trees - Insertion
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Challenge
Uphold
Weak Segment Tree Property
Zipping Segment Trees - Insertion

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Idea
Clear the “unzipped” path.
Zipping Segment Trees - Insertion

**Challenge**

Uphold Weak Segment Tree Property

**Idea**

Clear the “unzipped” path.
Zipping Segment Trees - Insertion

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- Uphold Weak Segment Tree Property

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Idea
Clear the “unzipped” path.

Correctness (Intuition)
Look at search paths along the cleared path.

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Zipping Segment Trees - Deletion
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What about search paths exiting to the right here?
Zipping Segment Trees - Deletion

What about search paths exiting to the right here?
Insertion Height

Main Idea

- We want to expect a balanced tree
- Insert node with prob. $\frac{1}{2}$ as leaf, with prob. $\frac{1}{4}$ at height 1, ...
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“Random” Variant
- Flip a coin until hitting “heads”
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“Random” Variant
- Flip a coin until hitting “heads”

“Hashing” Variant
- Hash the node’s value (or its memory address, or …)
- Use the bits as a stream of coin flips
- Advantage: Don’t need to store the rank at the node!
Experimental Results

1. Create tree with \( n \) random intervals (\( x \) axis)
2. Insert \( k \) new random intervals
3. \( y \) axis: Time for step 2 divided by \( k \)

![Graph showing insertion times for different tree structures](image)

- Red-Black
- Weight-Balanced
- Zip (Hashing)
- Zip (Random)
Experimental Results

1. Create tree with $n$ random intervals ($x$ axis)
2. Insert $k$ new random intervals
3. $y$ axis: Time for step 2 divided by $k$

[B, Wagner. SEA 2020.]

Red-Black
Weight-Balanced
Zip (Hashing)
Zip (Random)

[B, Wagner. ALENEX 2020.]
Experimental Results

1. Create tree with \( n \) random intervals \((x \text{ axis})\)
2. Insert \( k \) new random intervals
3. \( y \text{ axis}: \text{Time for step 2 divided by } k \)

**Graph:**
- **Red-Black**
- **Weight-Balanced**
- **Zip (Hashing)**
- **Zip (Random)**

**Axes:**
- **x axis:** Tree Size \((\times 10^4)\)
- **y axis:** Time (\(\mu\text{s}\))
Experimental Results

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- Red-Black
- Weight-Balanced
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[B, Wagner. SEA 2020.]
Conclusion
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- Most efficient choice for deletions and moves
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- Next step: Tuning Zip Trees!