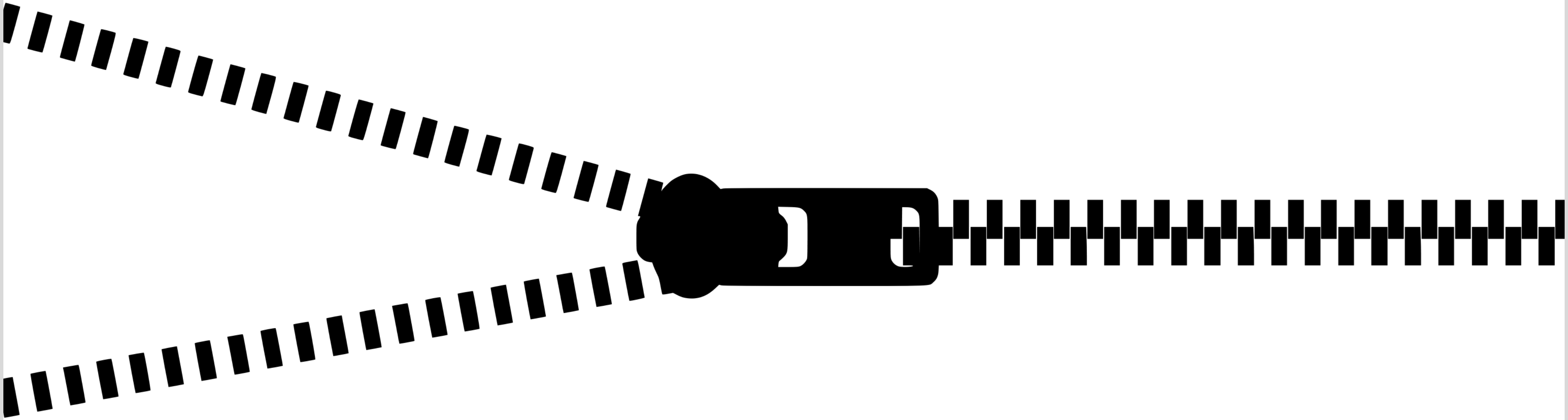
Karlsruhe Institute of Technology

Zipping Segment Trees

SEA 2020 · 18.6.2020

Lukas Barth, Dorothea Wagner

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP



Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]

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Stabbing Query

Given a set of intervals $\mathcal{M}$ and a point p, find all intervals $I \in \mathcal{M}$ with $p \in I$.

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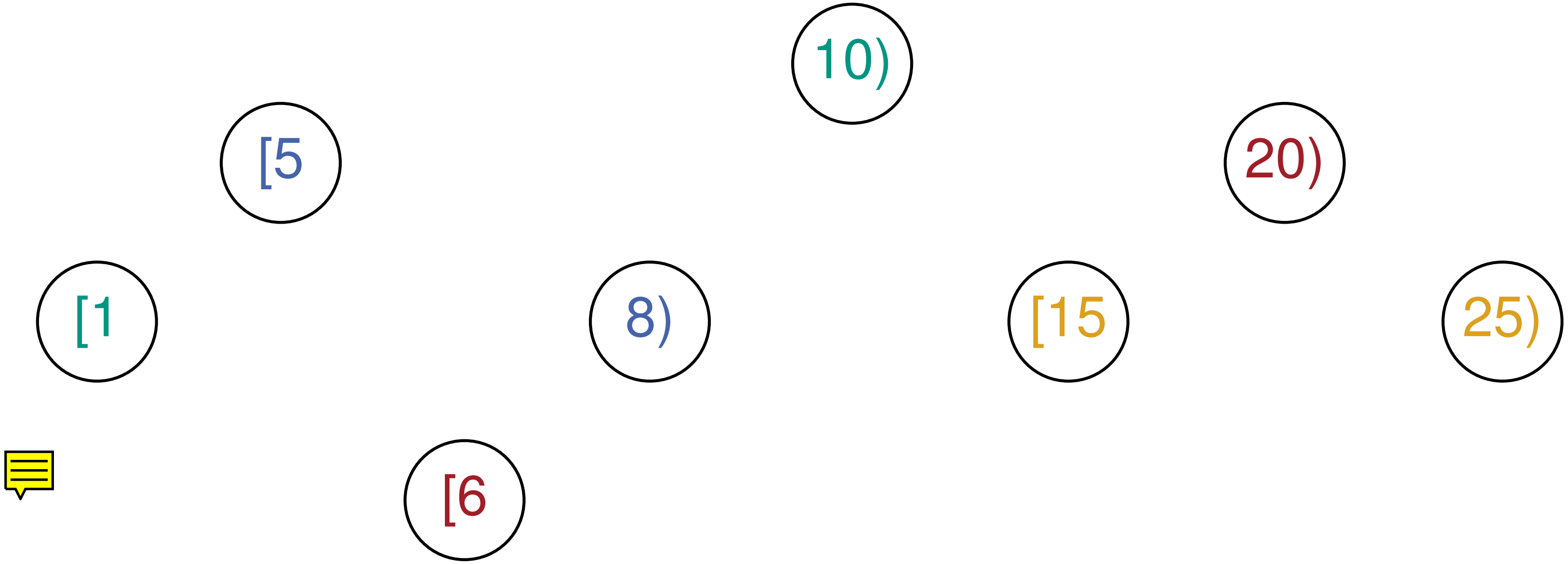

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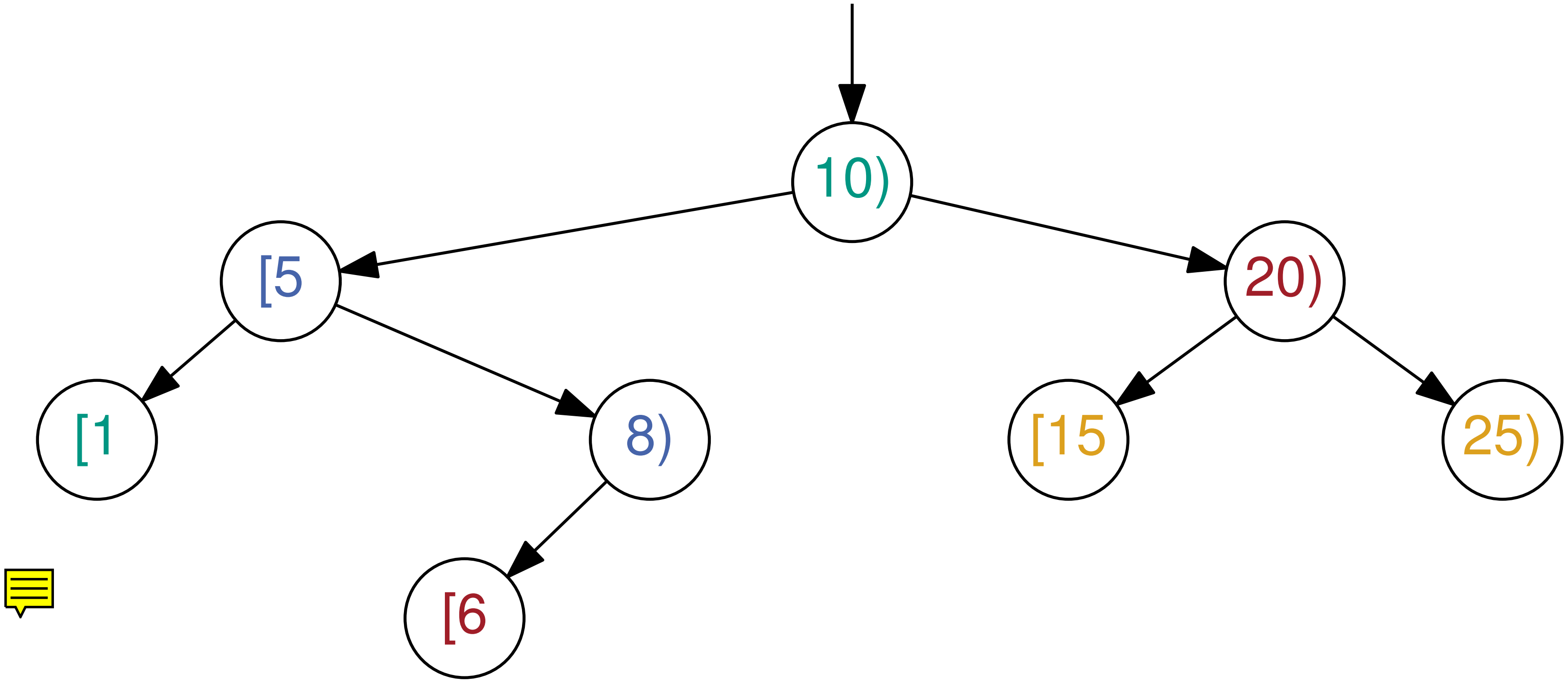

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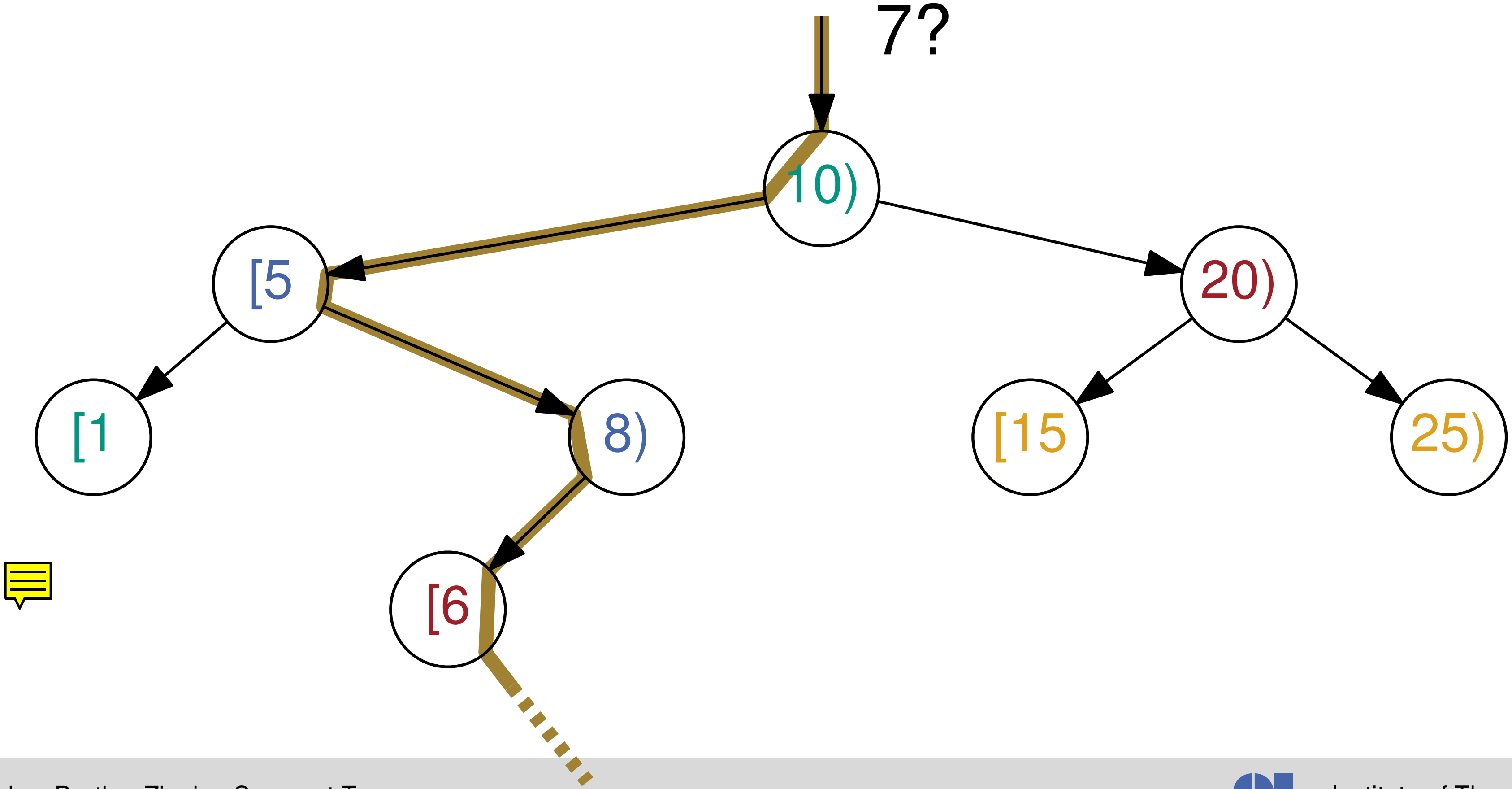

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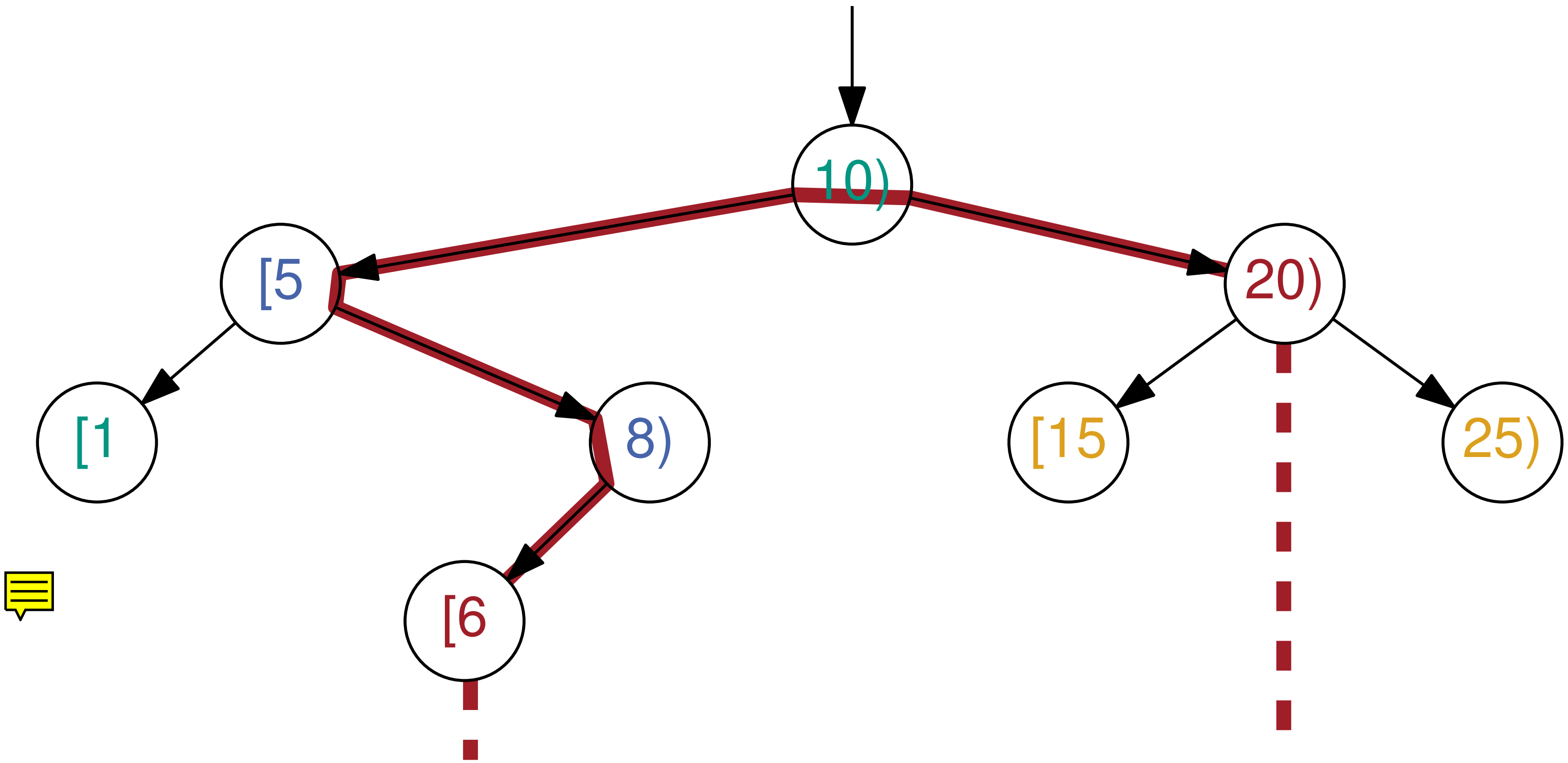

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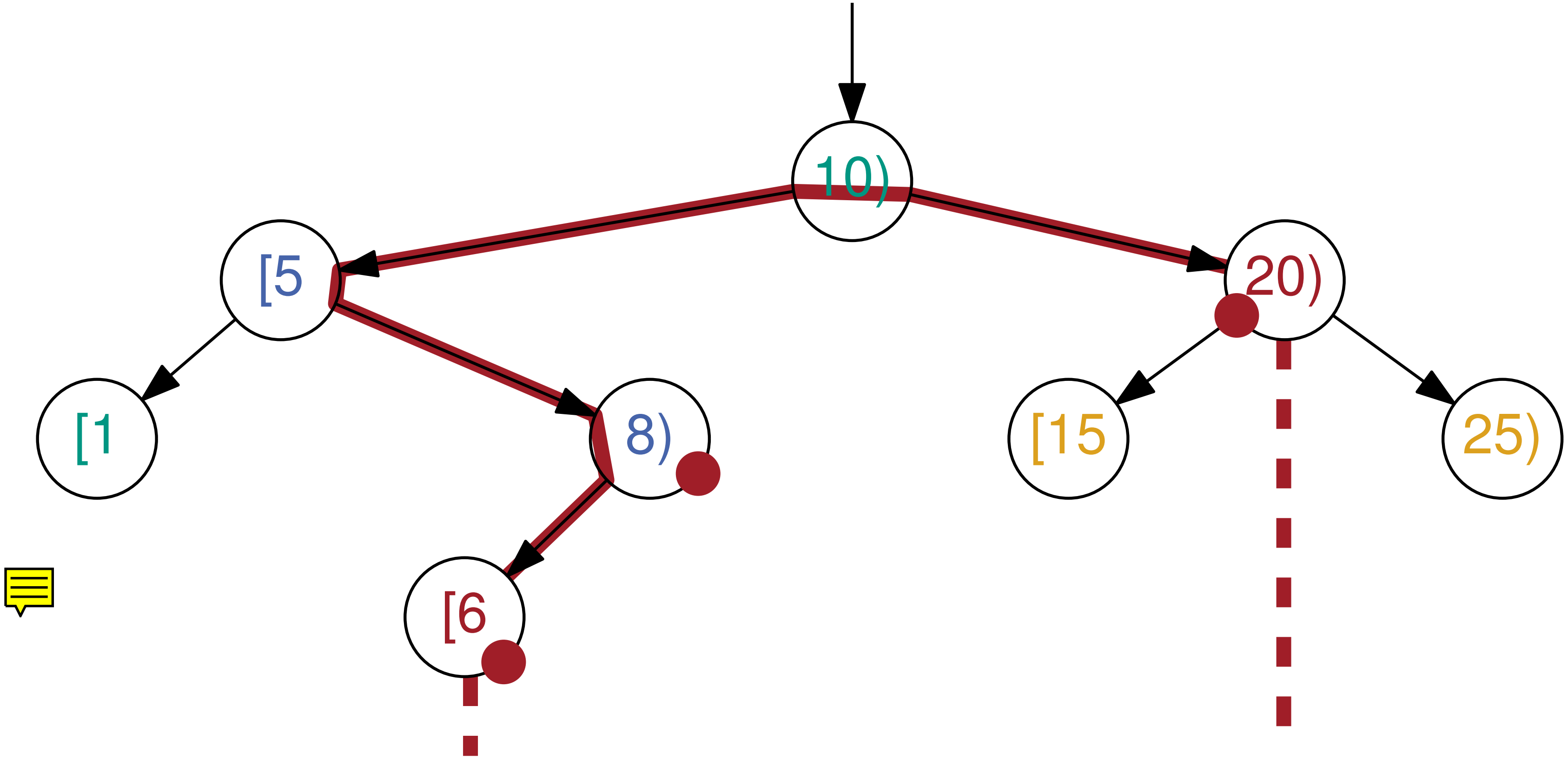

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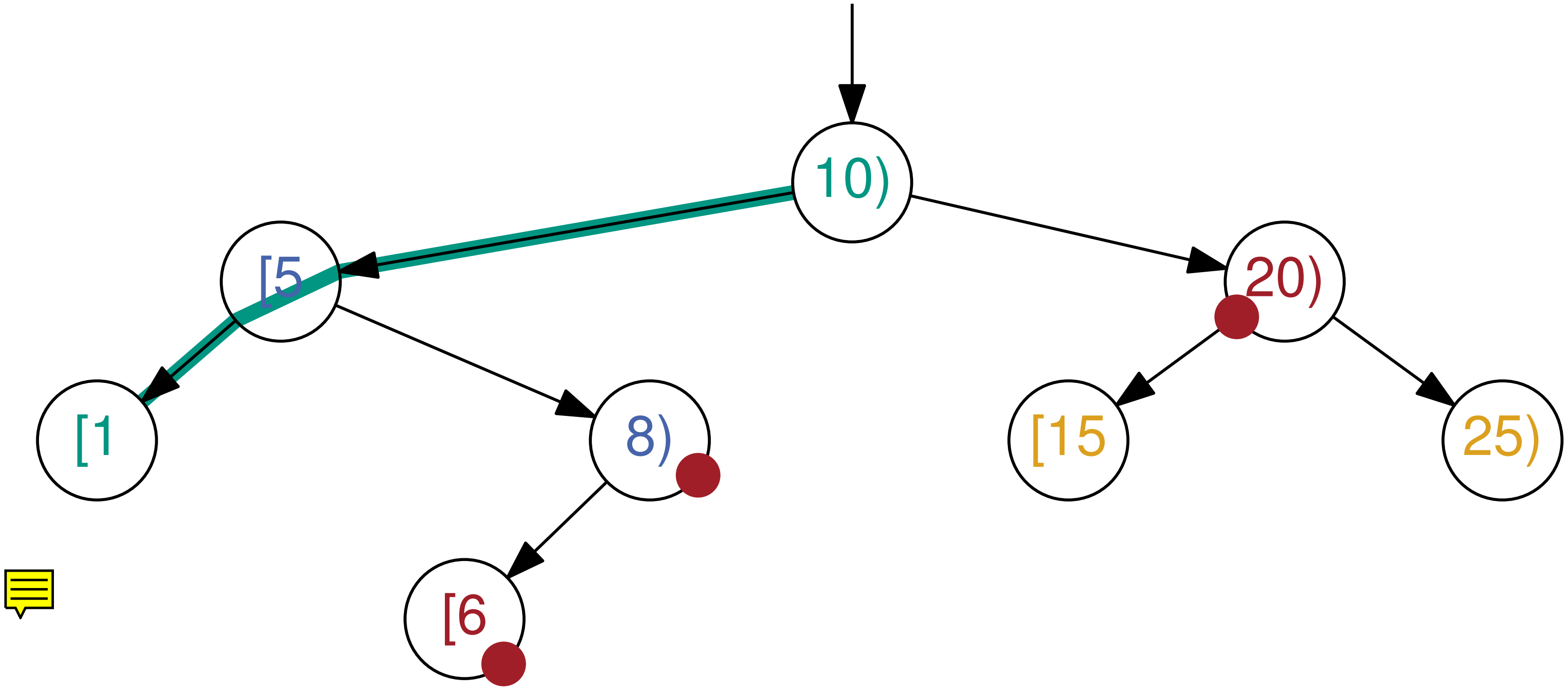

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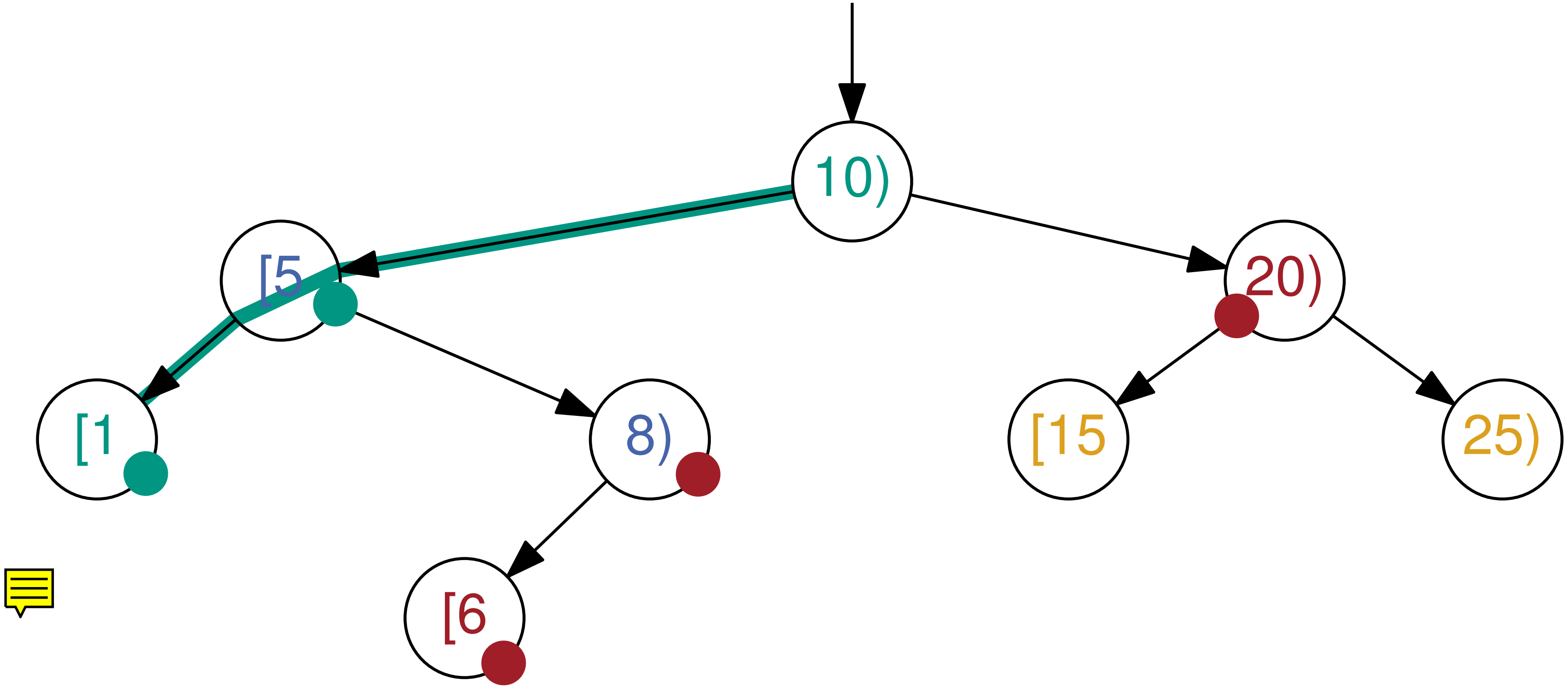

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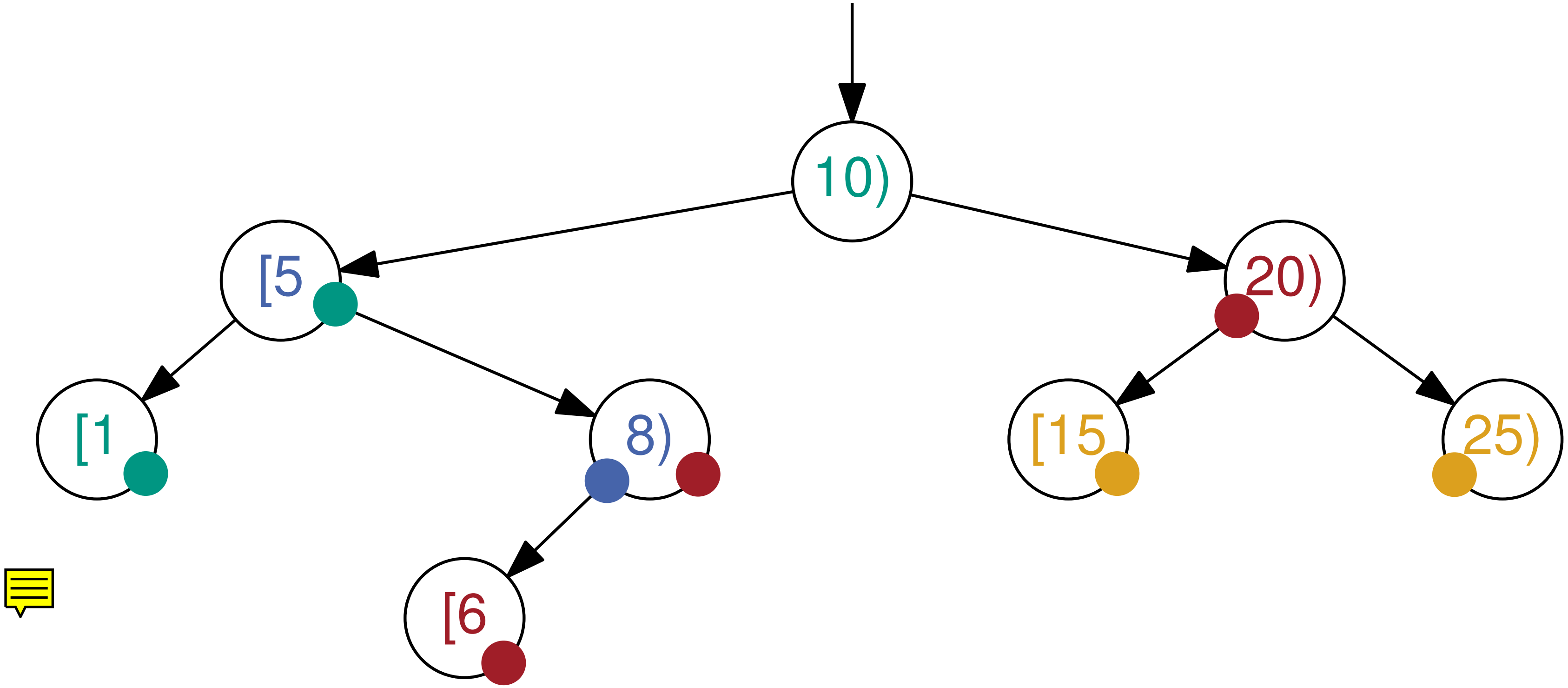

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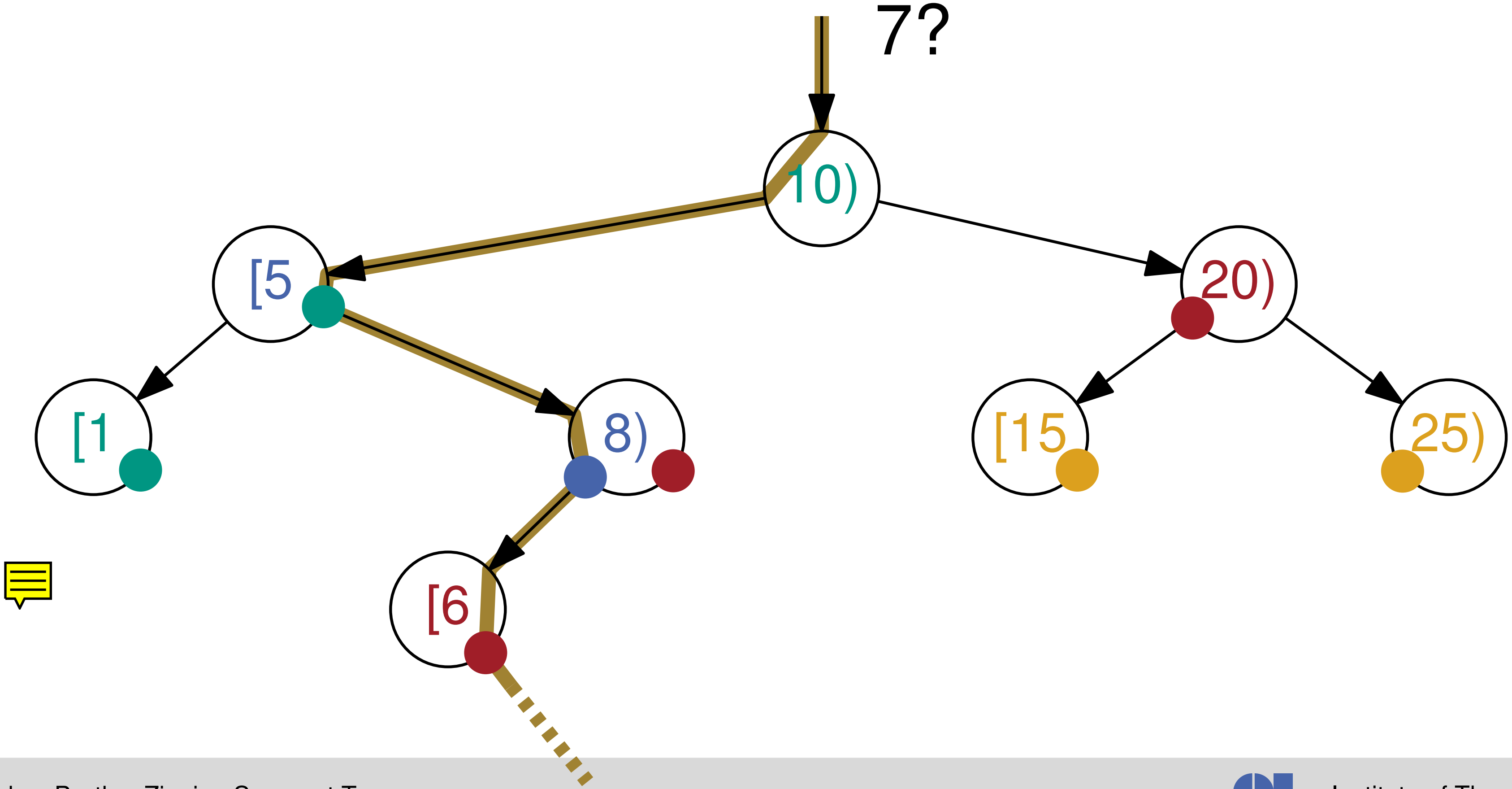

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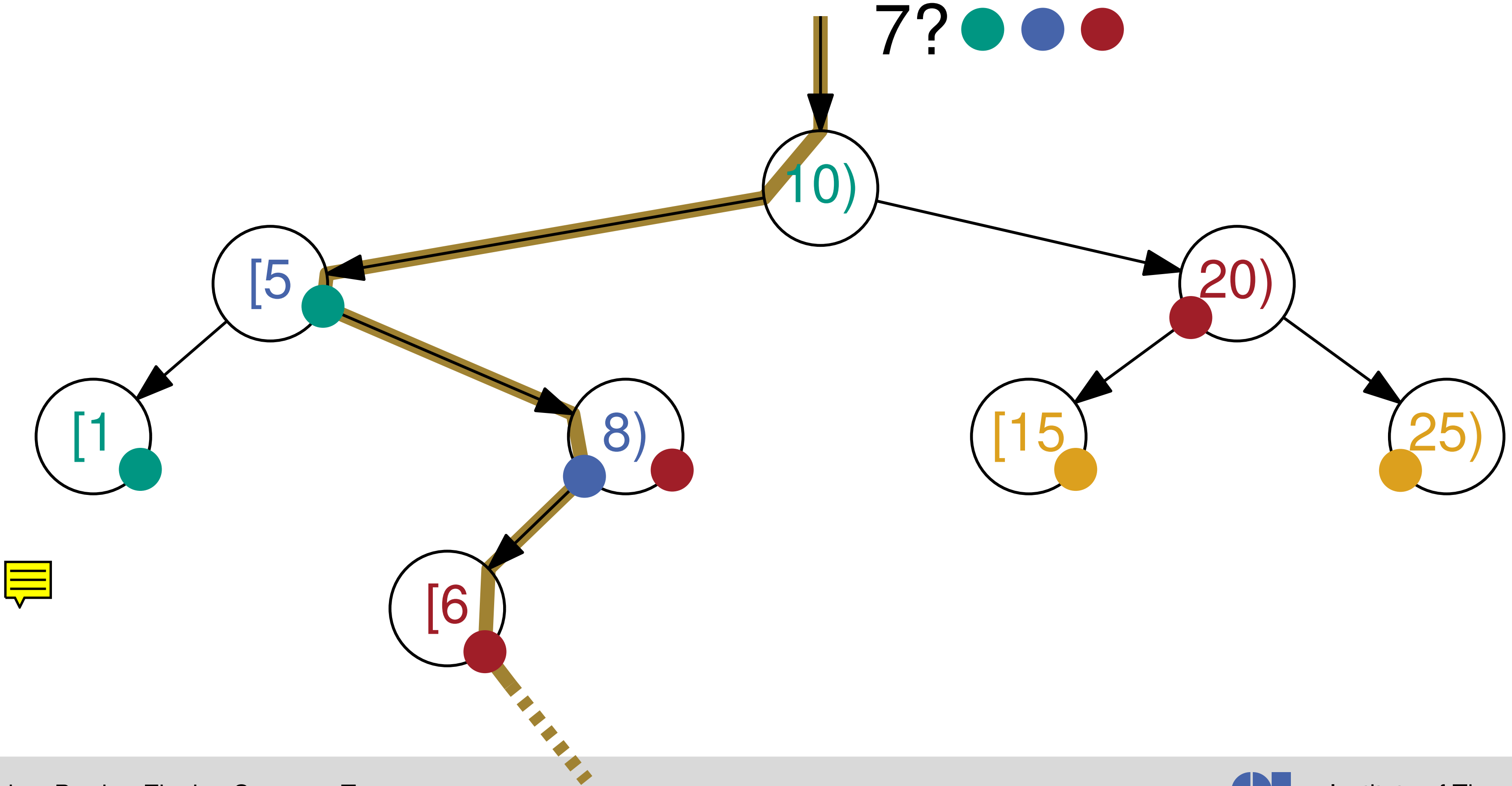

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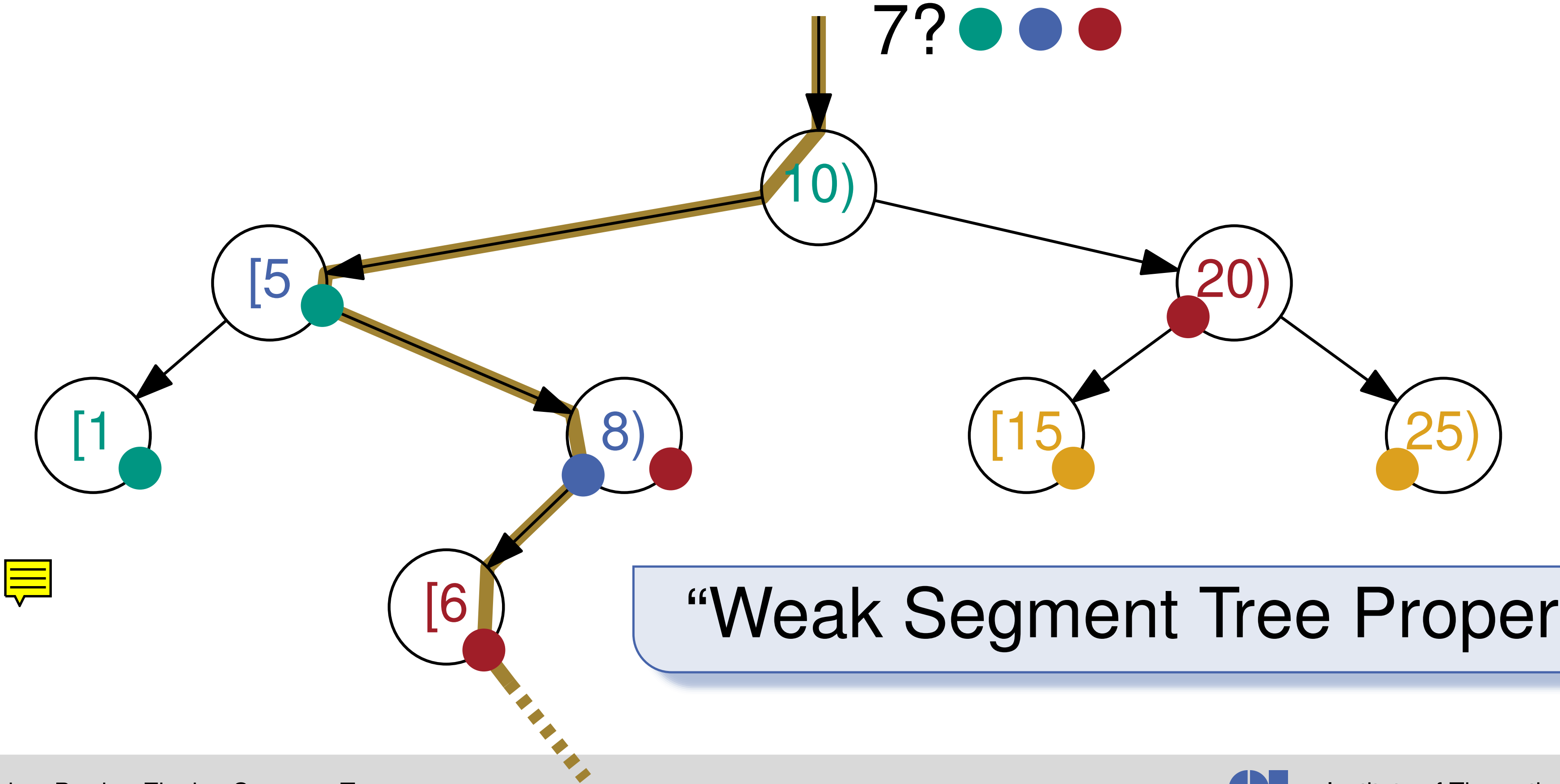

“Weak Segment Tree Property”

Dynamic Segment Trees

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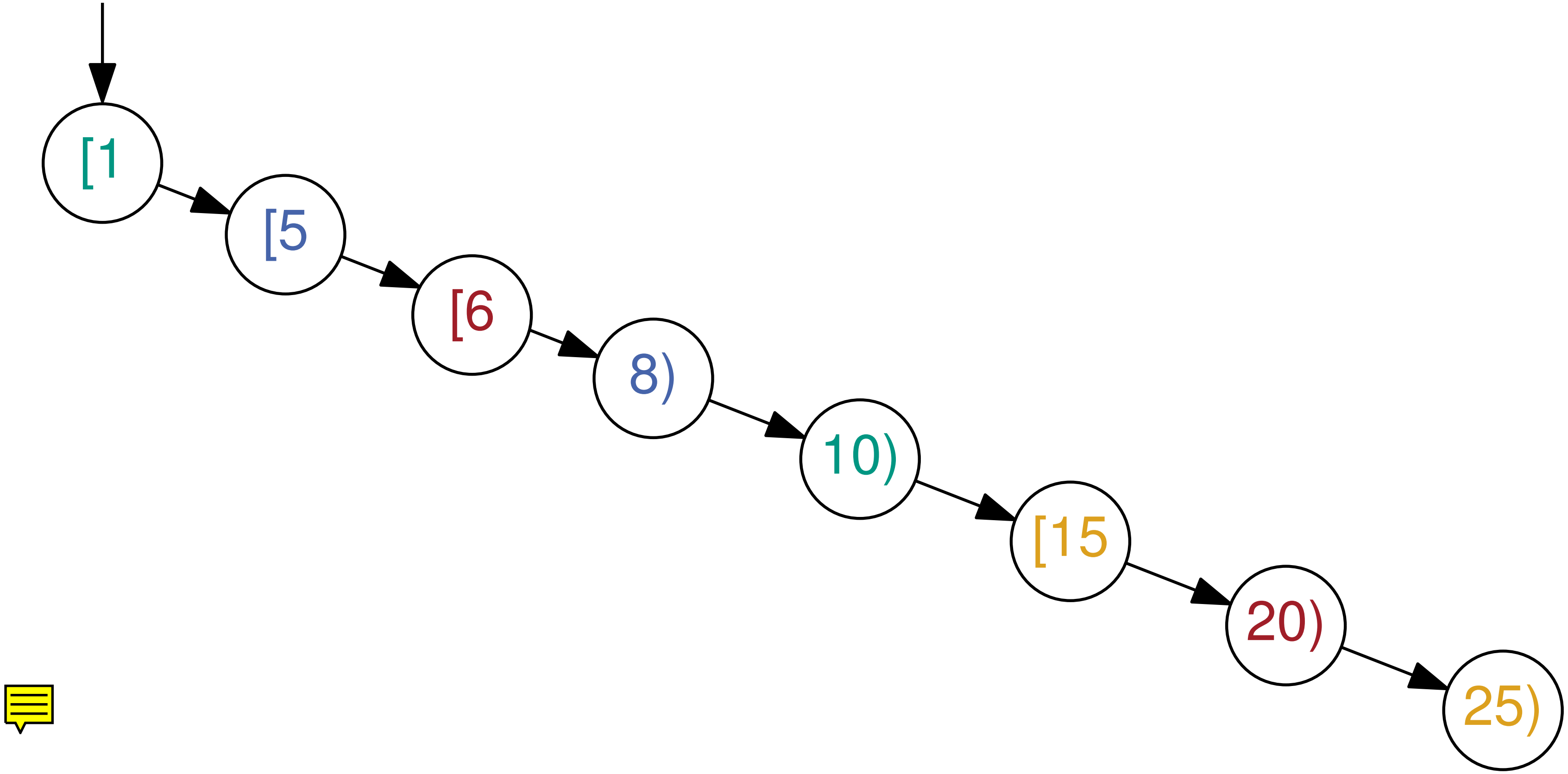

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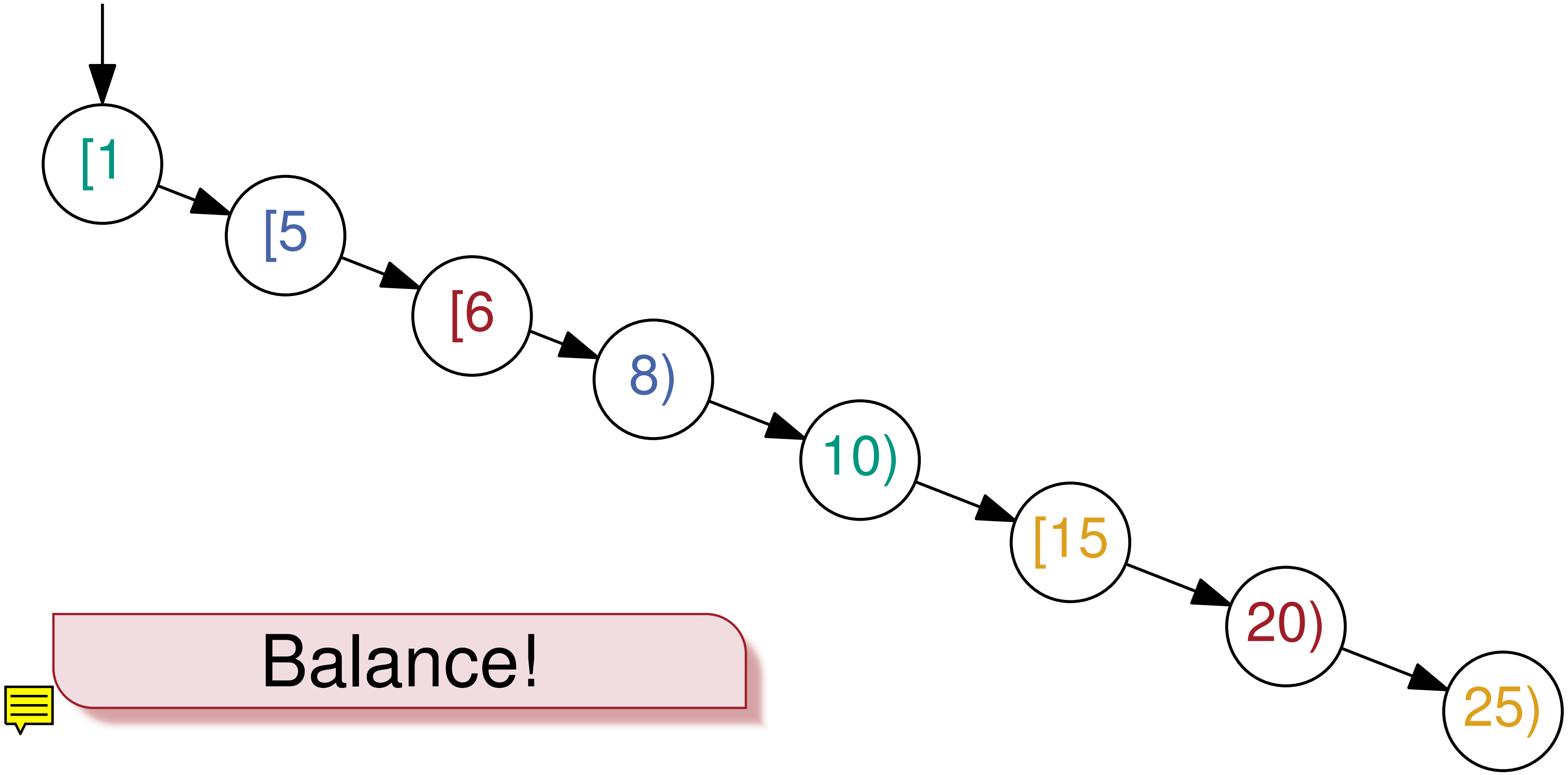
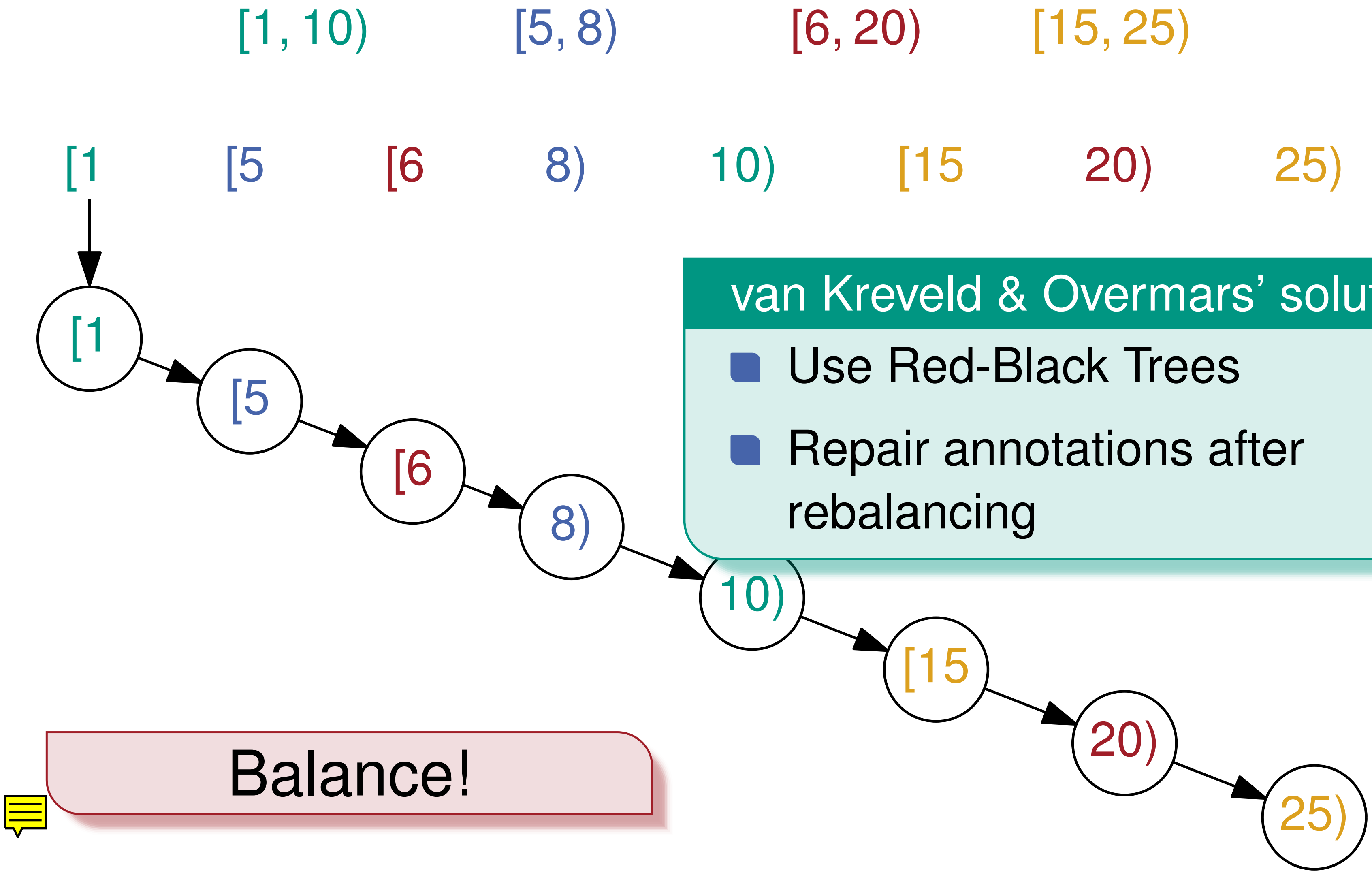


Balance!

Dynamic Segment Trees [van Kreveld, Overmars, JACM 1993]



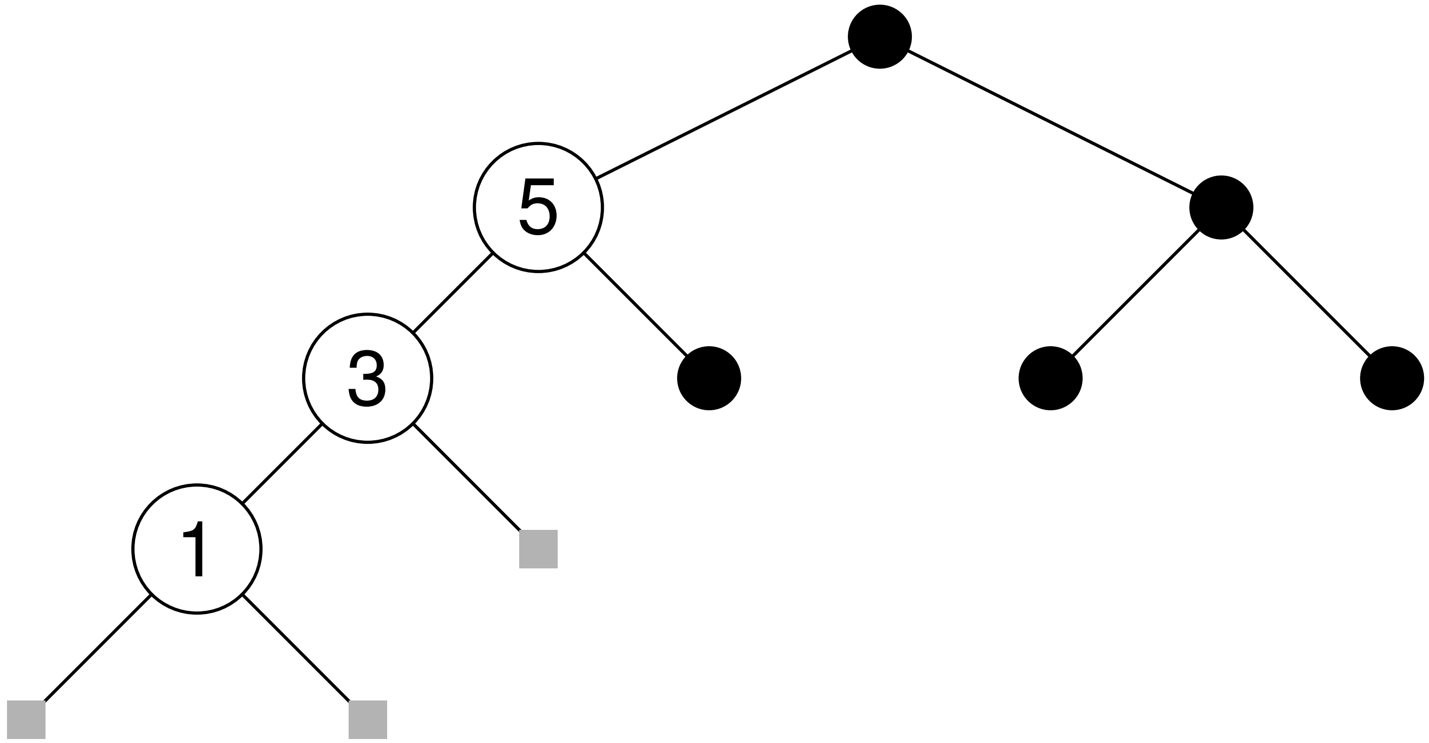
van Kreveld & Overmars' solution

- Use Red-Black Trees
- Repair annotations after rebalancing

Balance!

Zip Trees — Insertion

[Tarjan et al. WADS 2019.]



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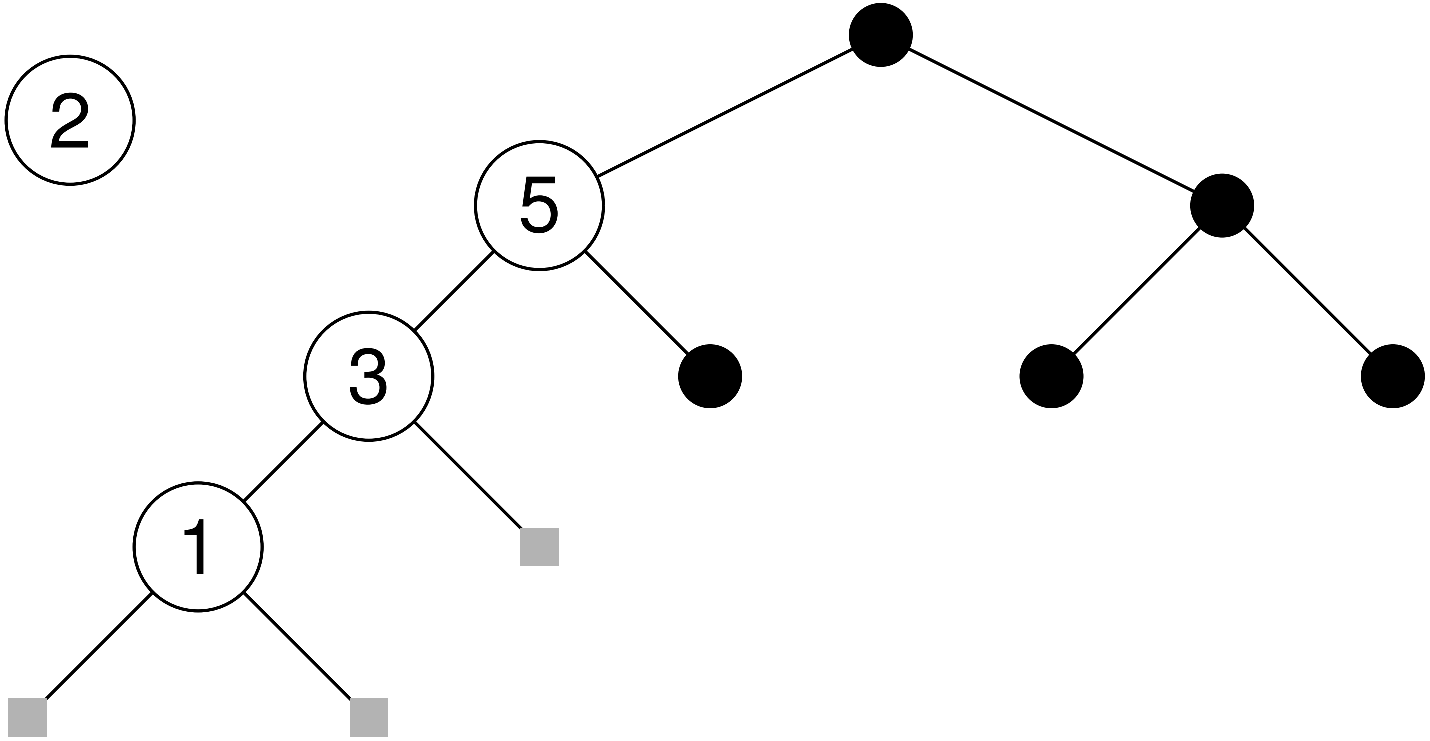

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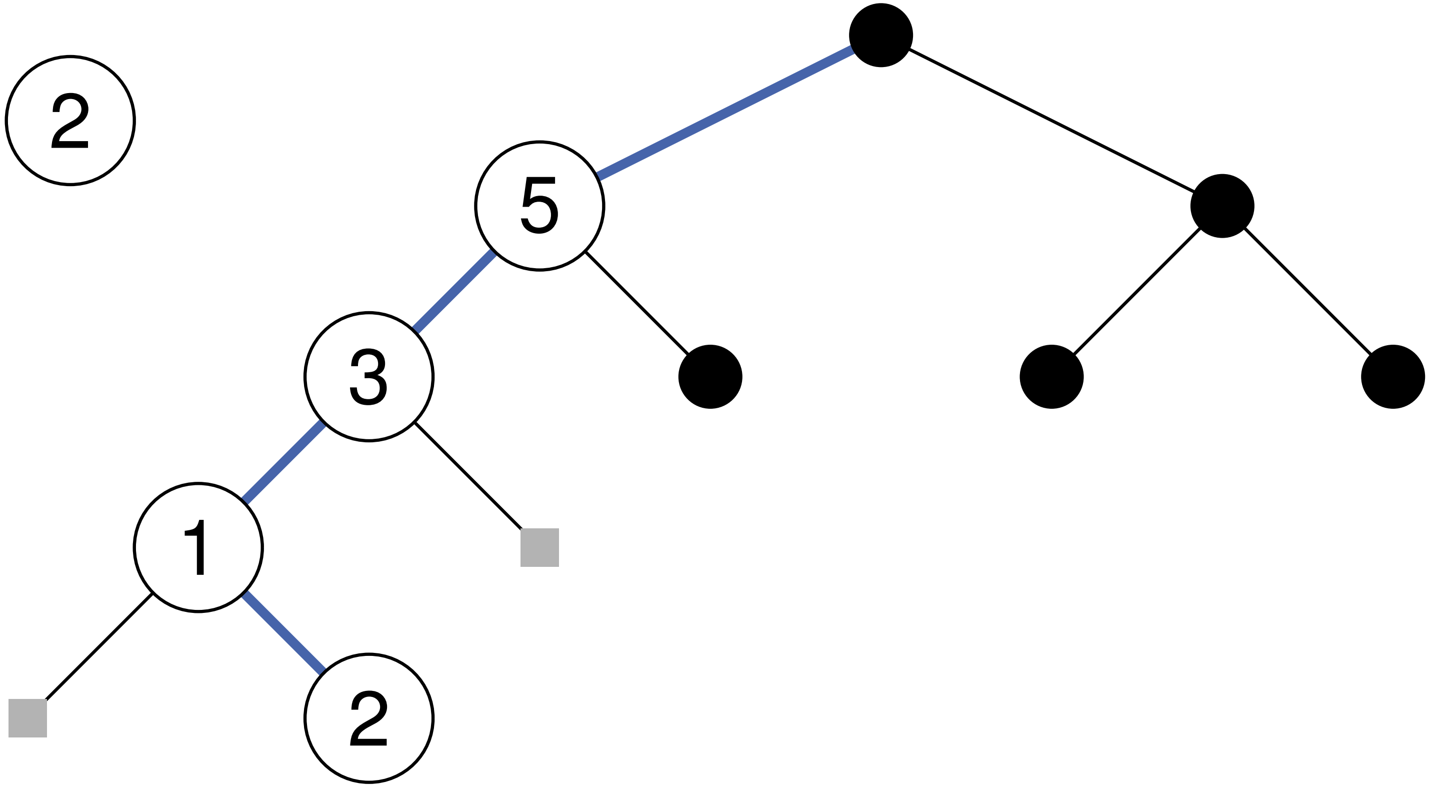

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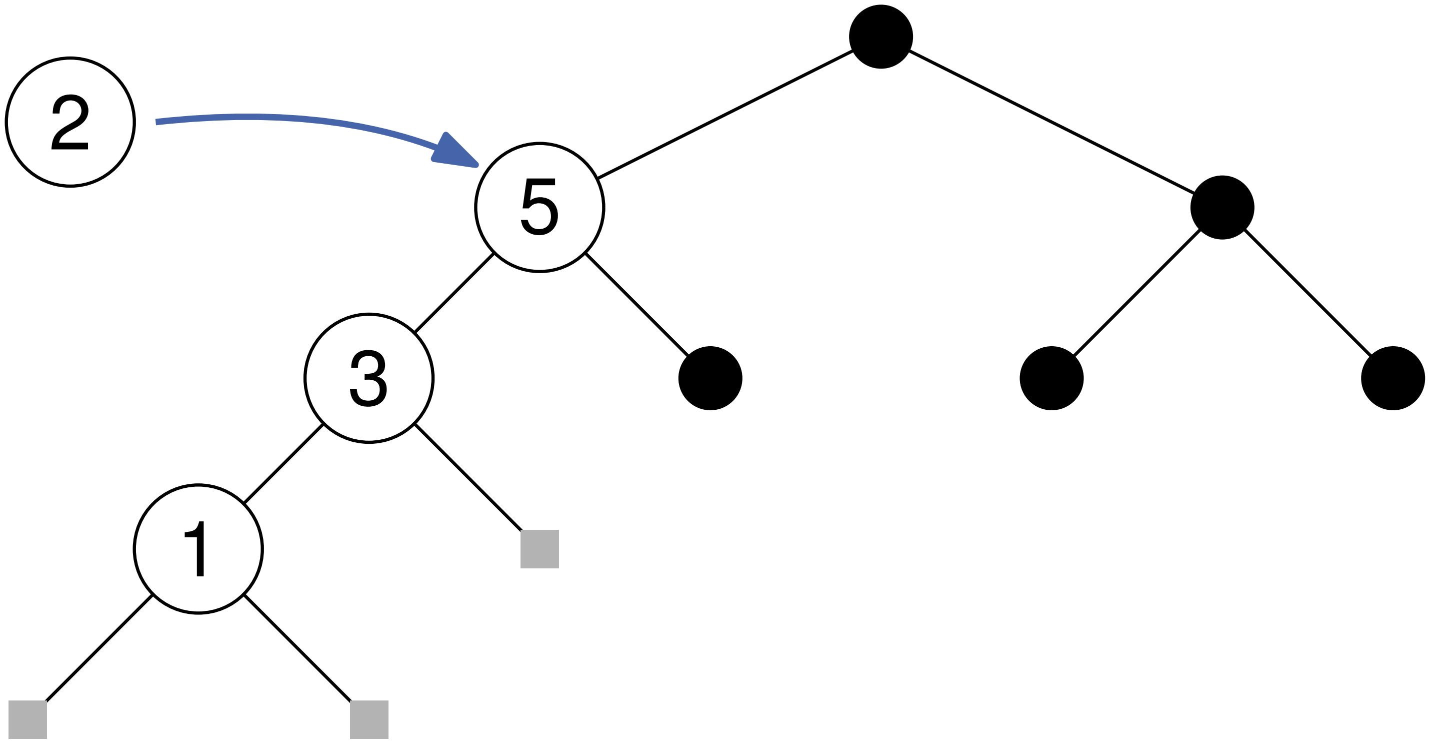

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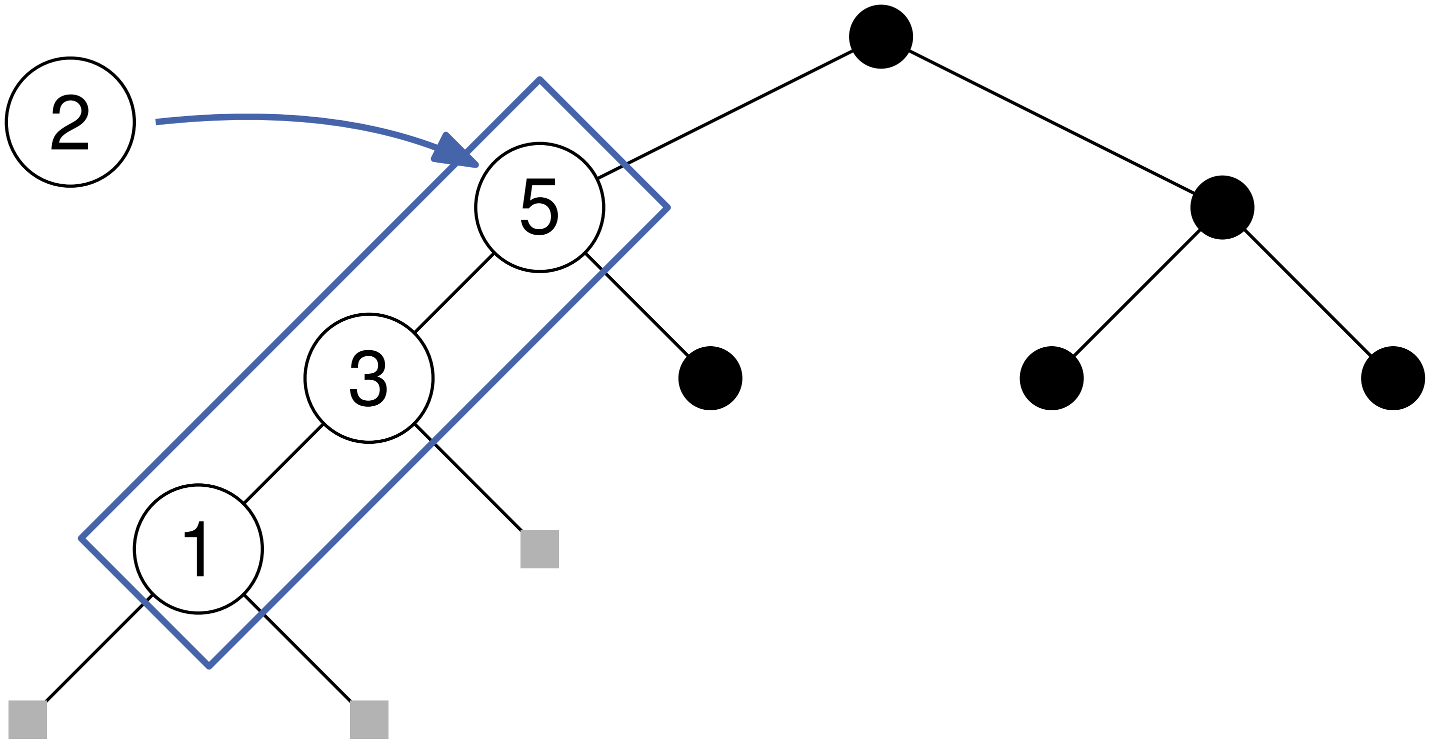

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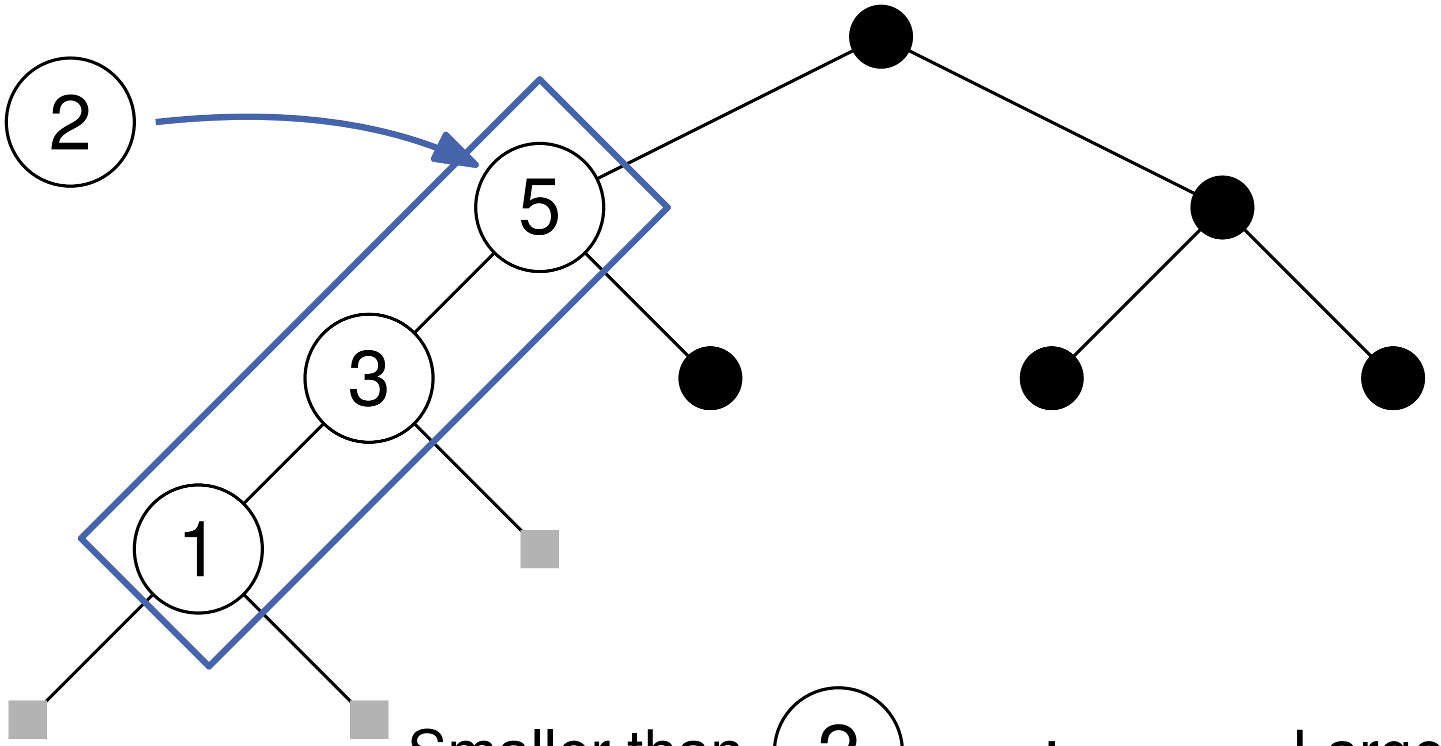

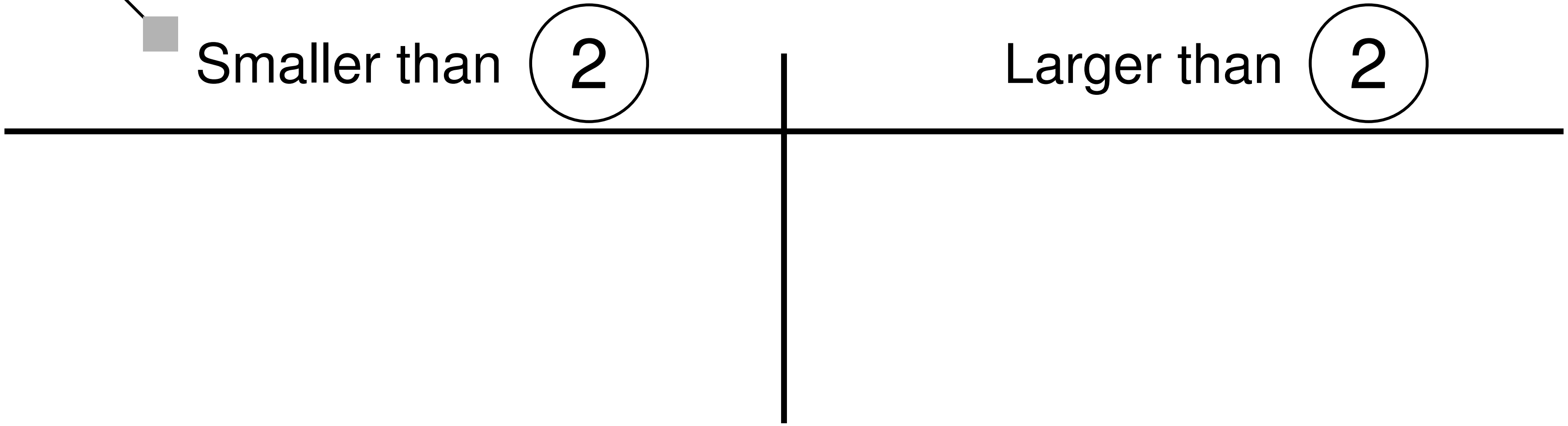

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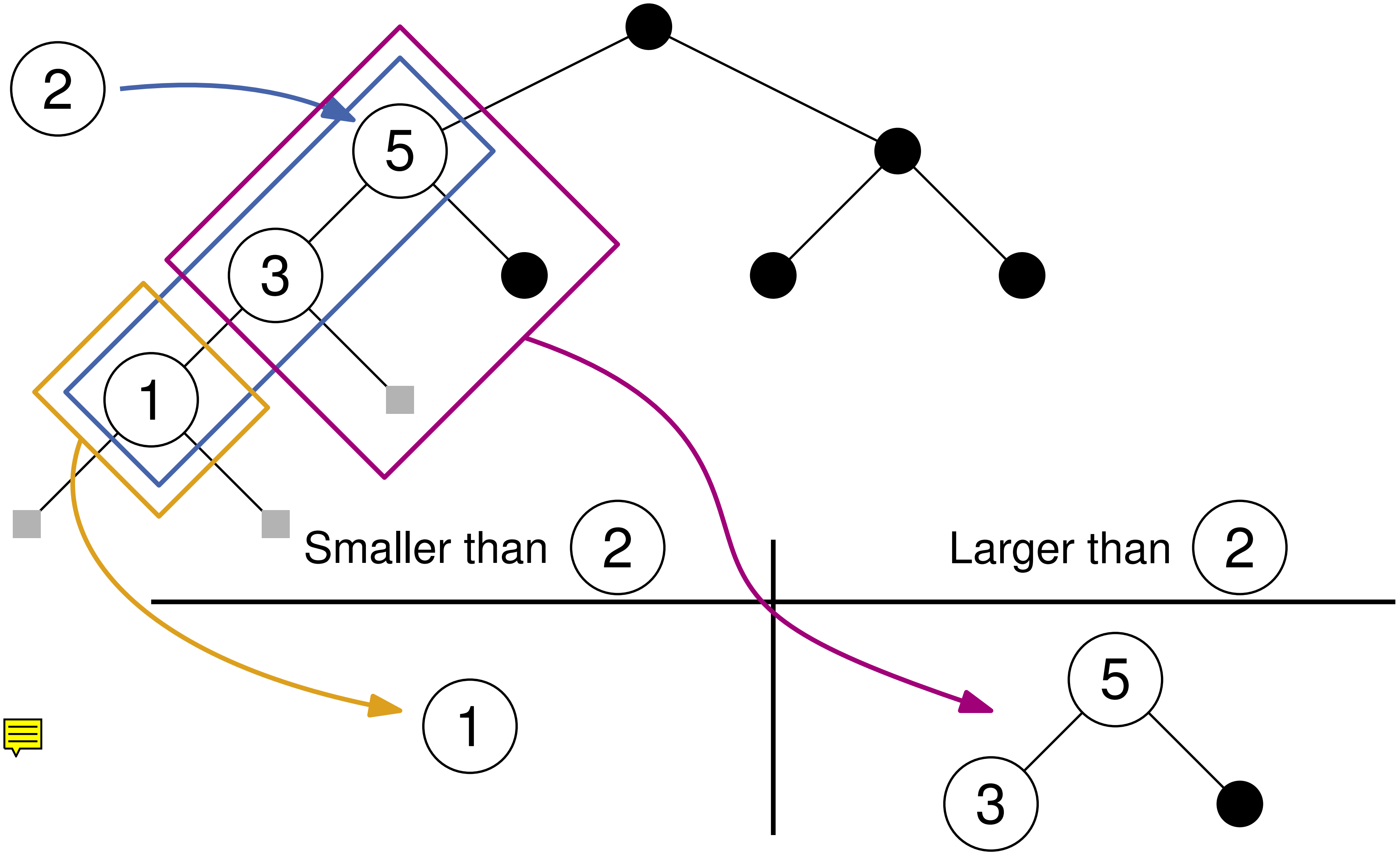

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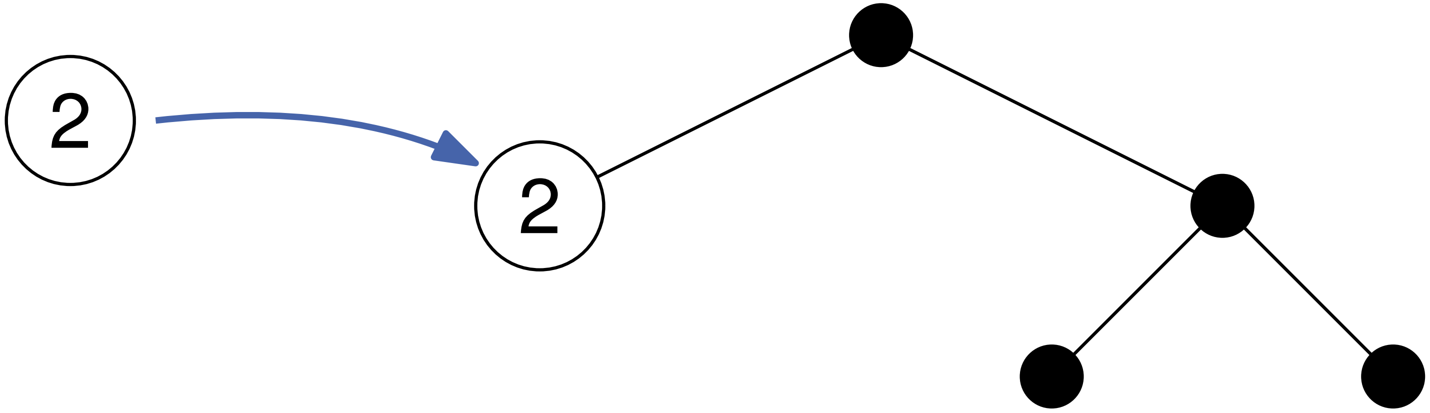

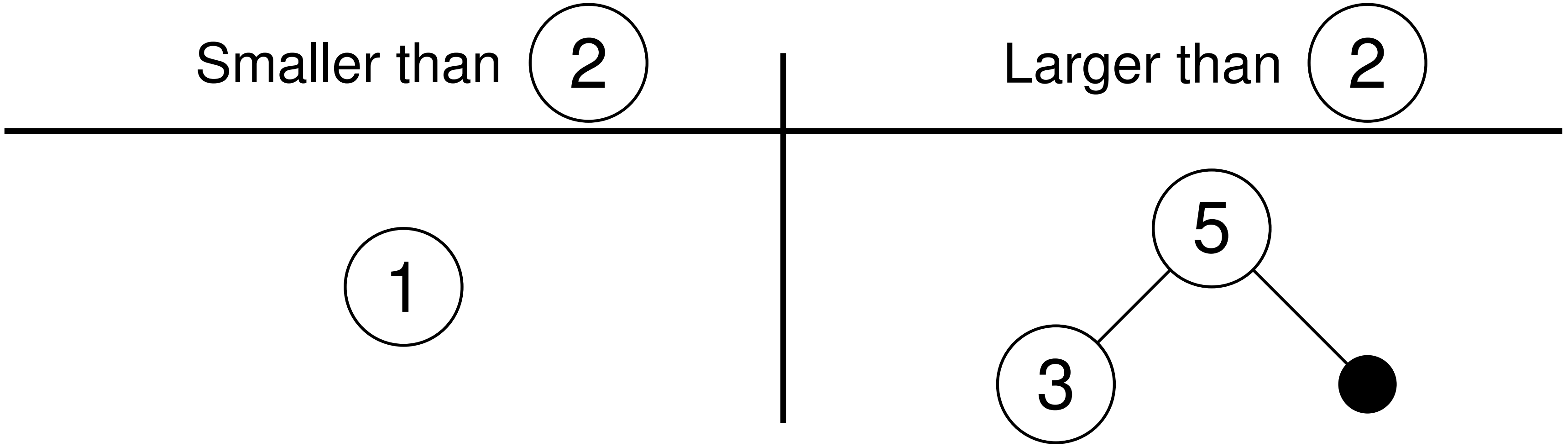
Smaller than 2

Larger than 2

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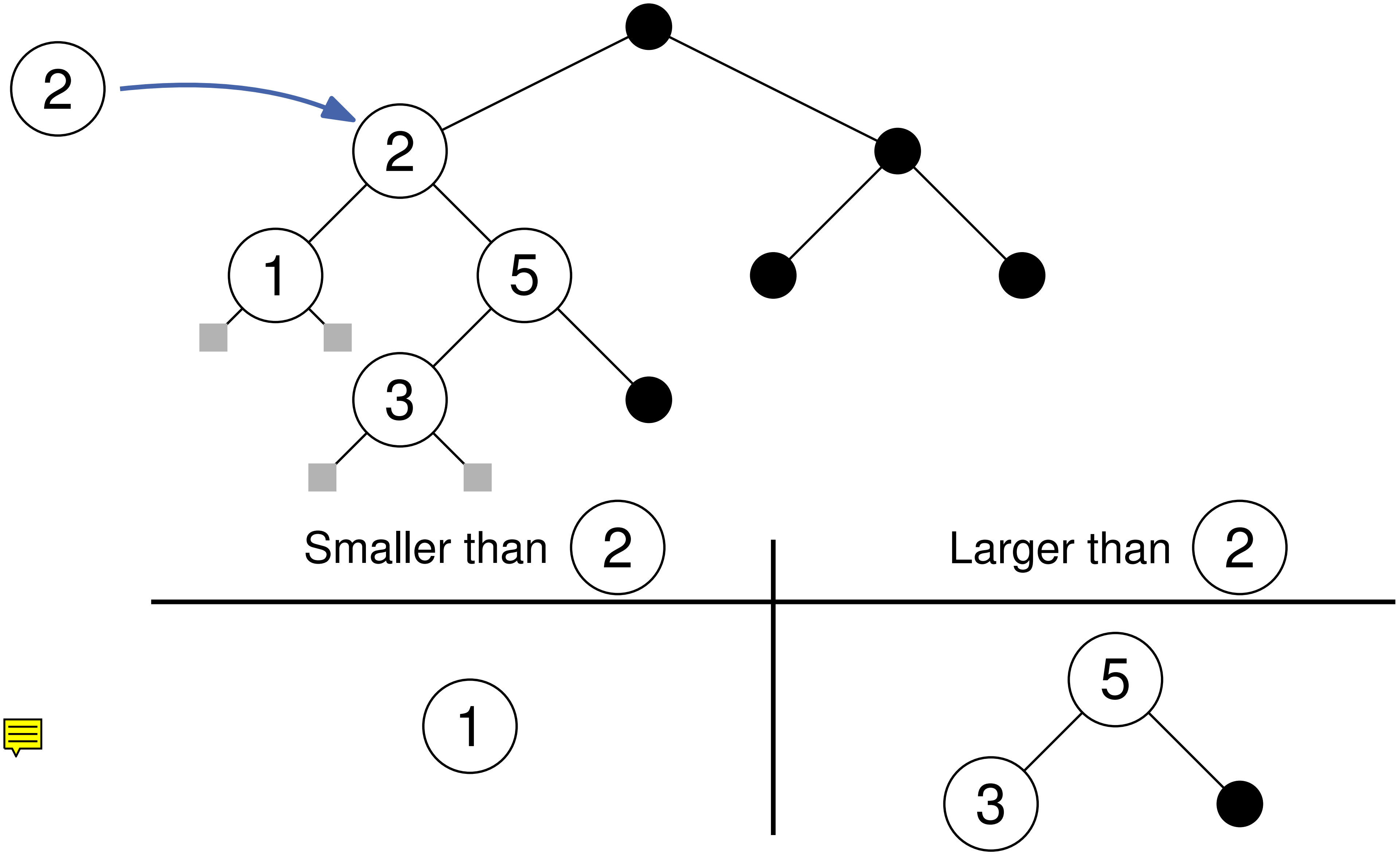

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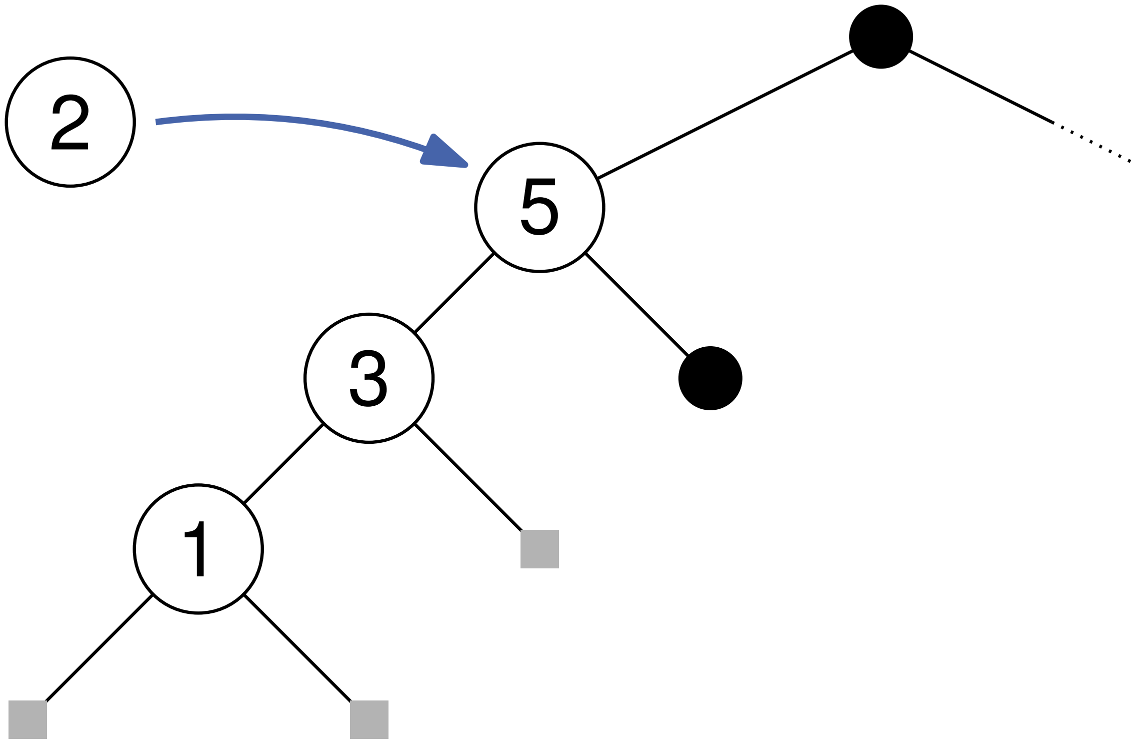

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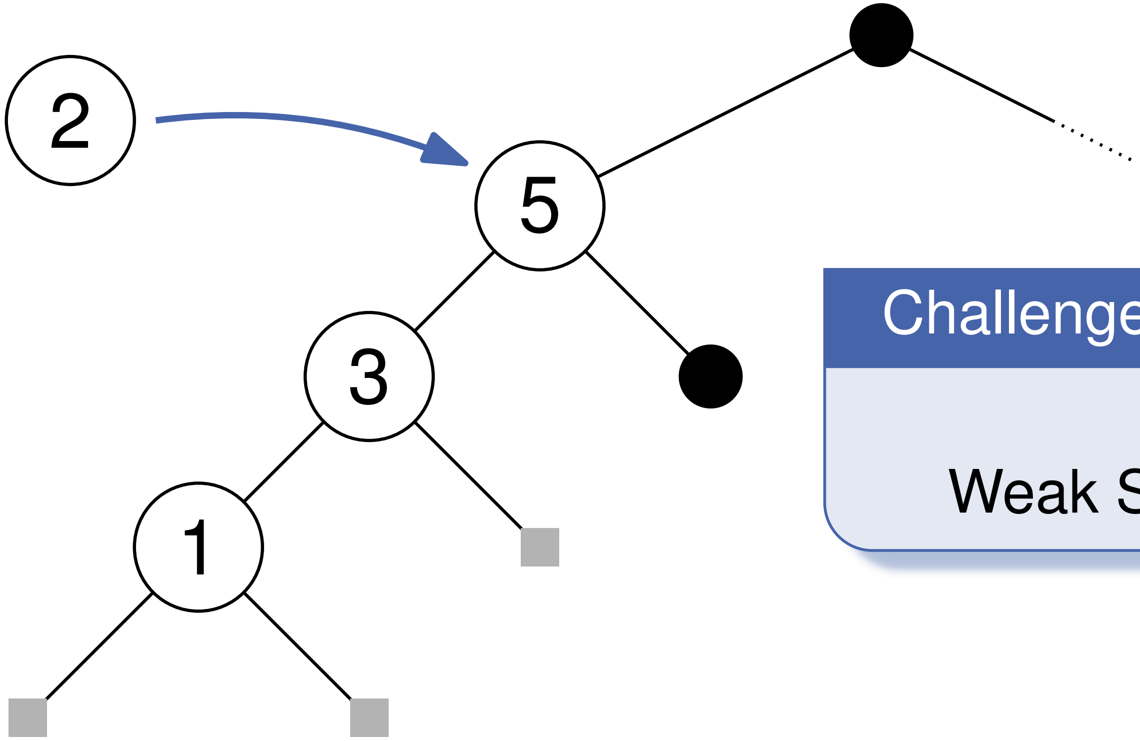

Challenge

Uphold
Weak Segment Tree Property

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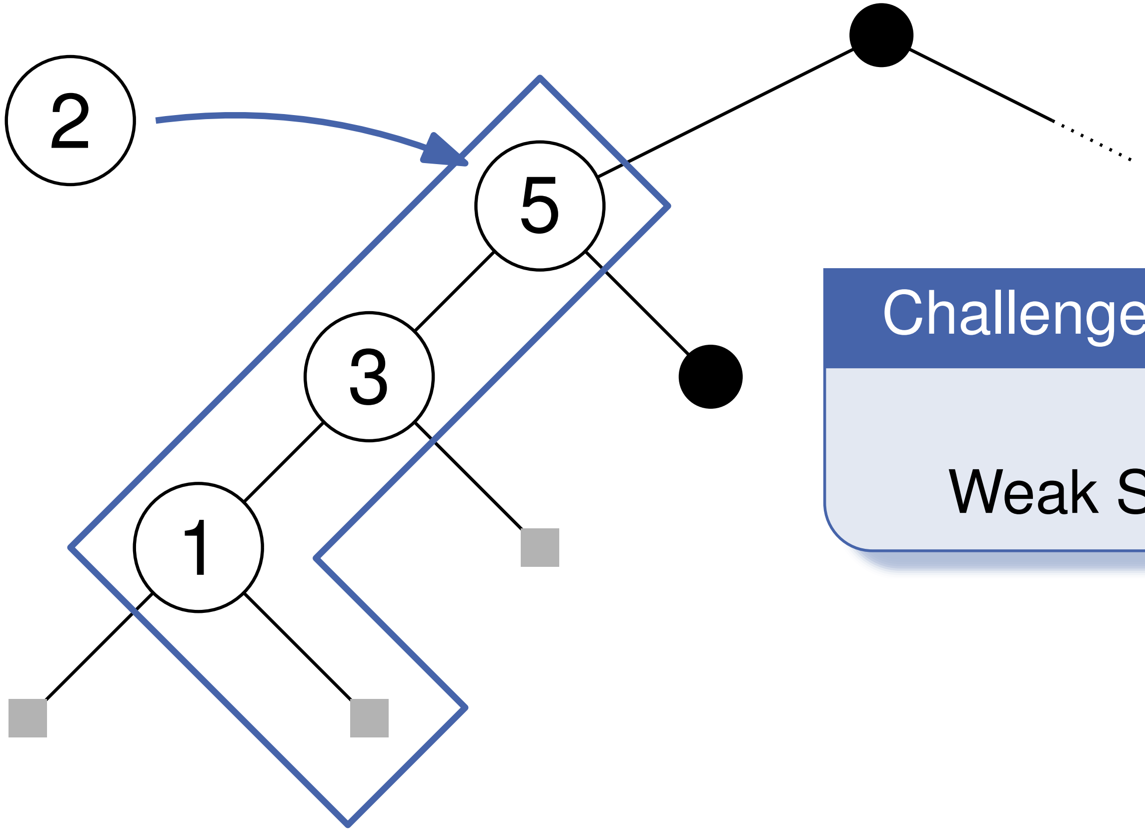

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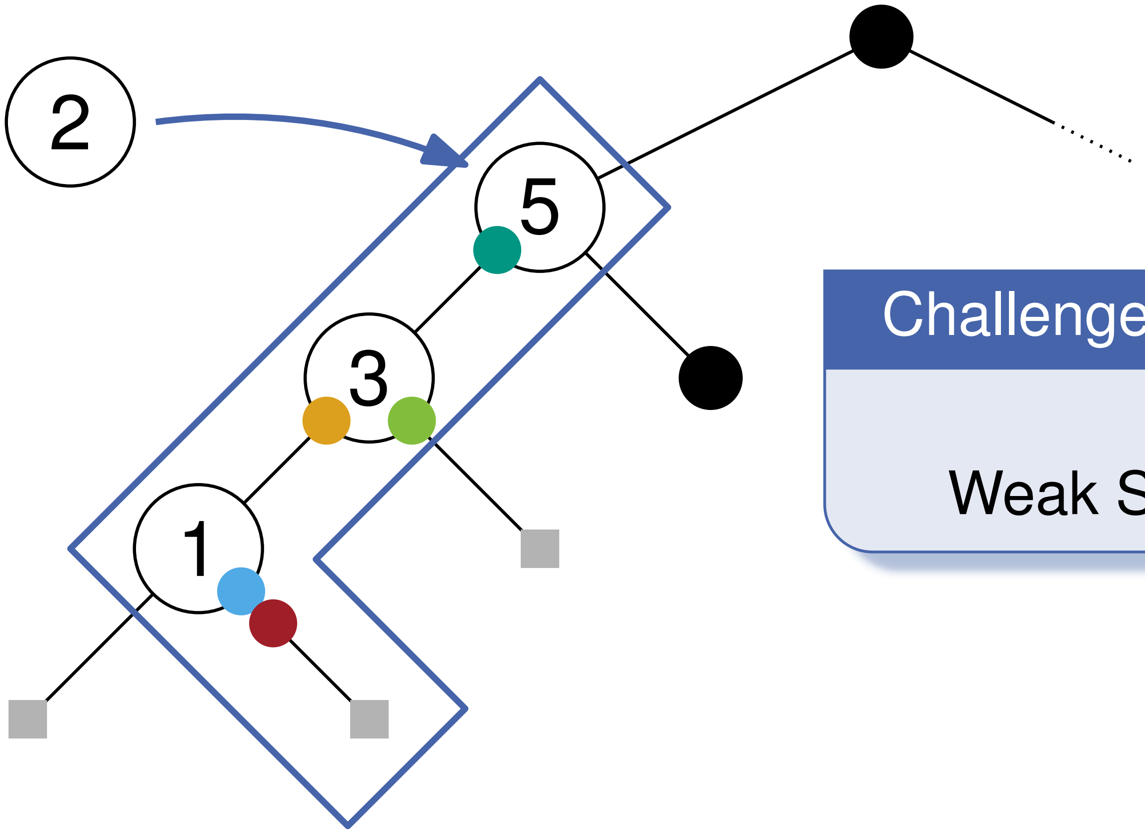

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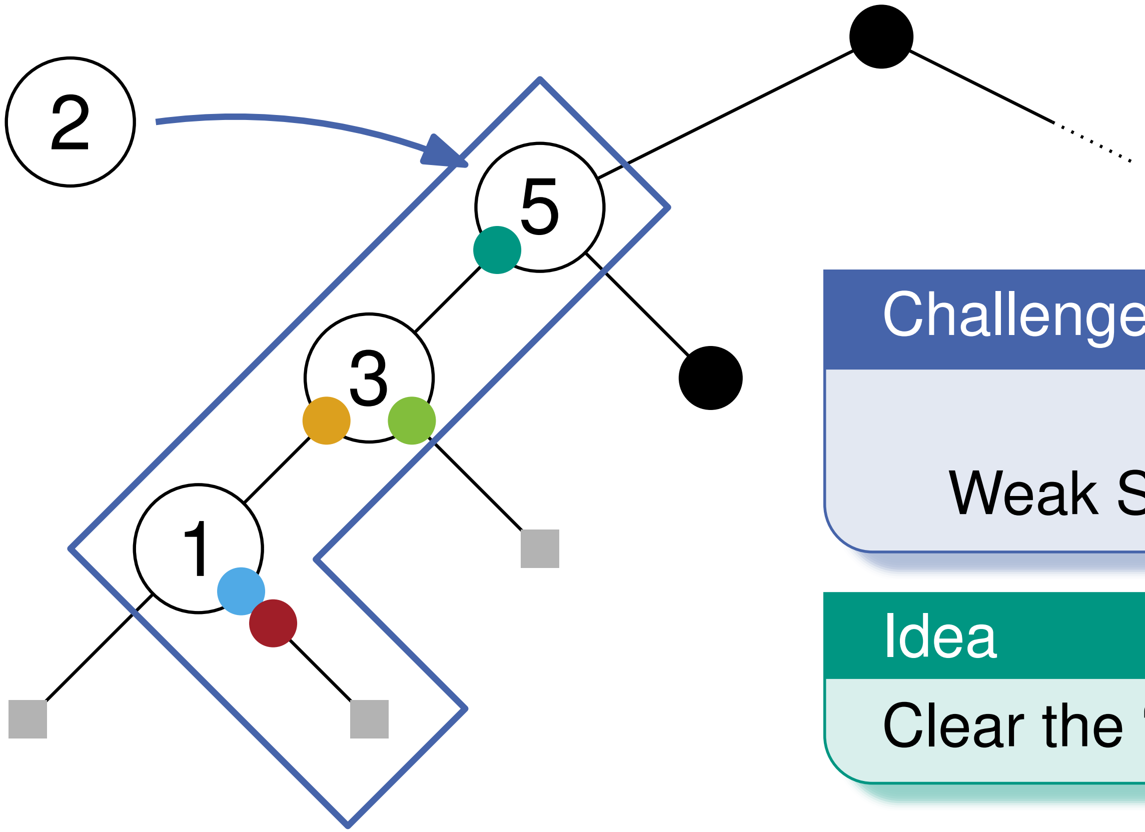

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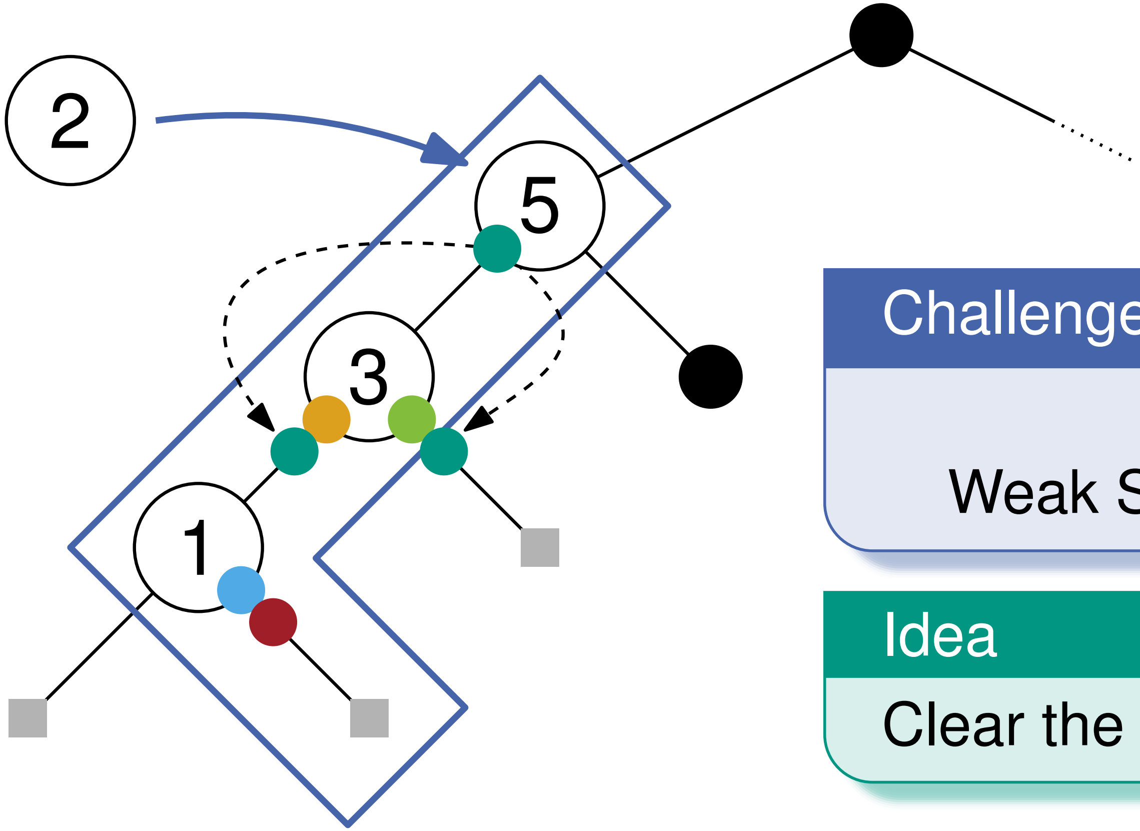
Idea

Clear the “unzipped” path.

Zipping Segment Trees - Insertion



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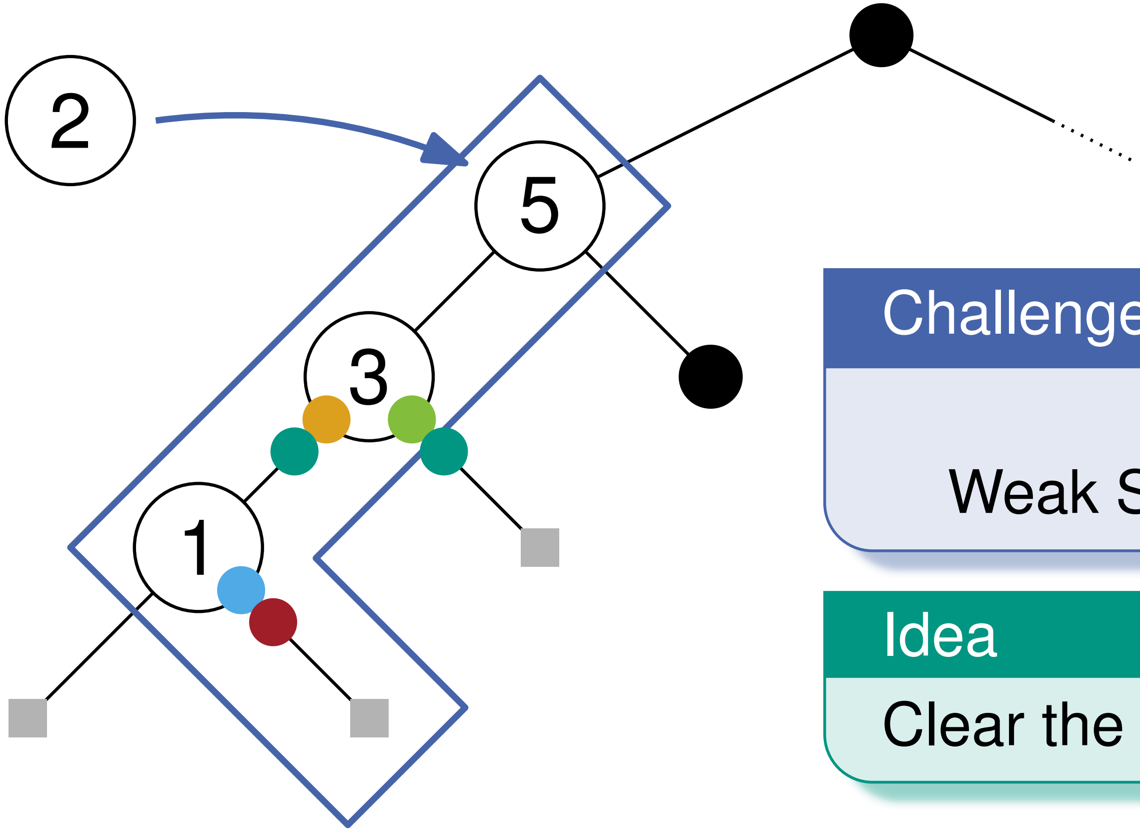

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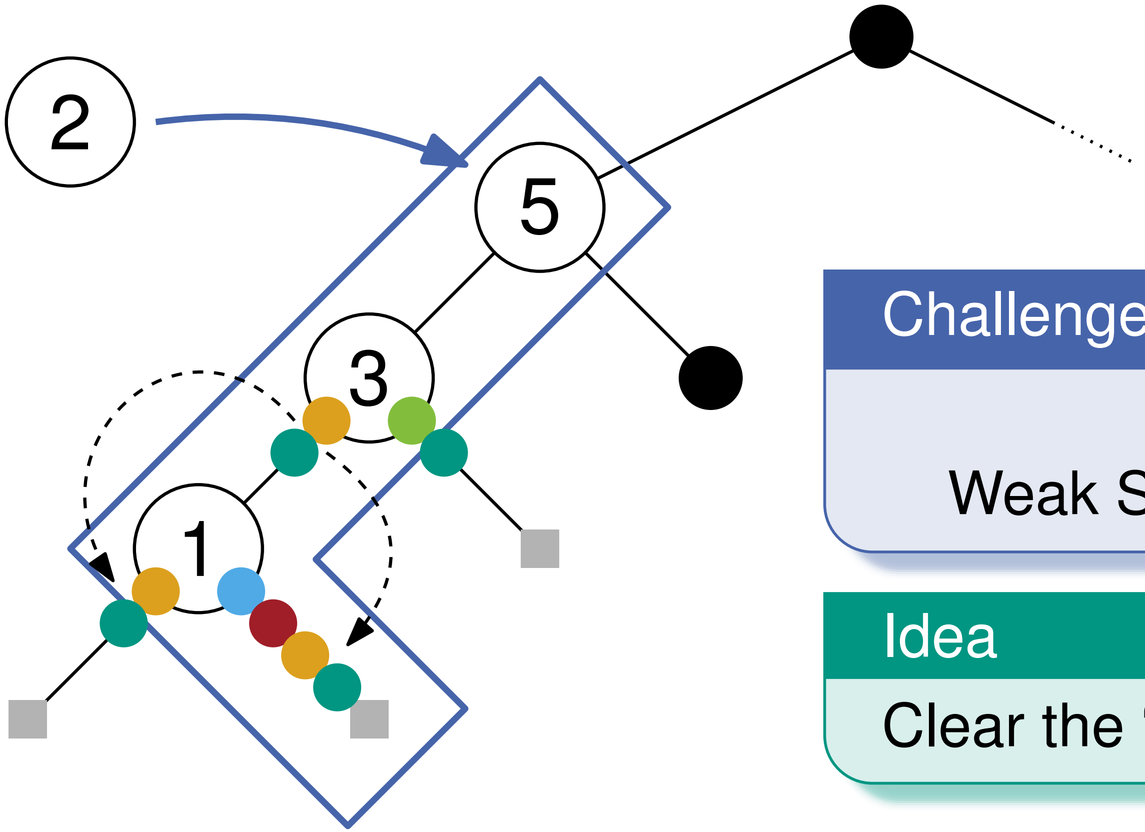

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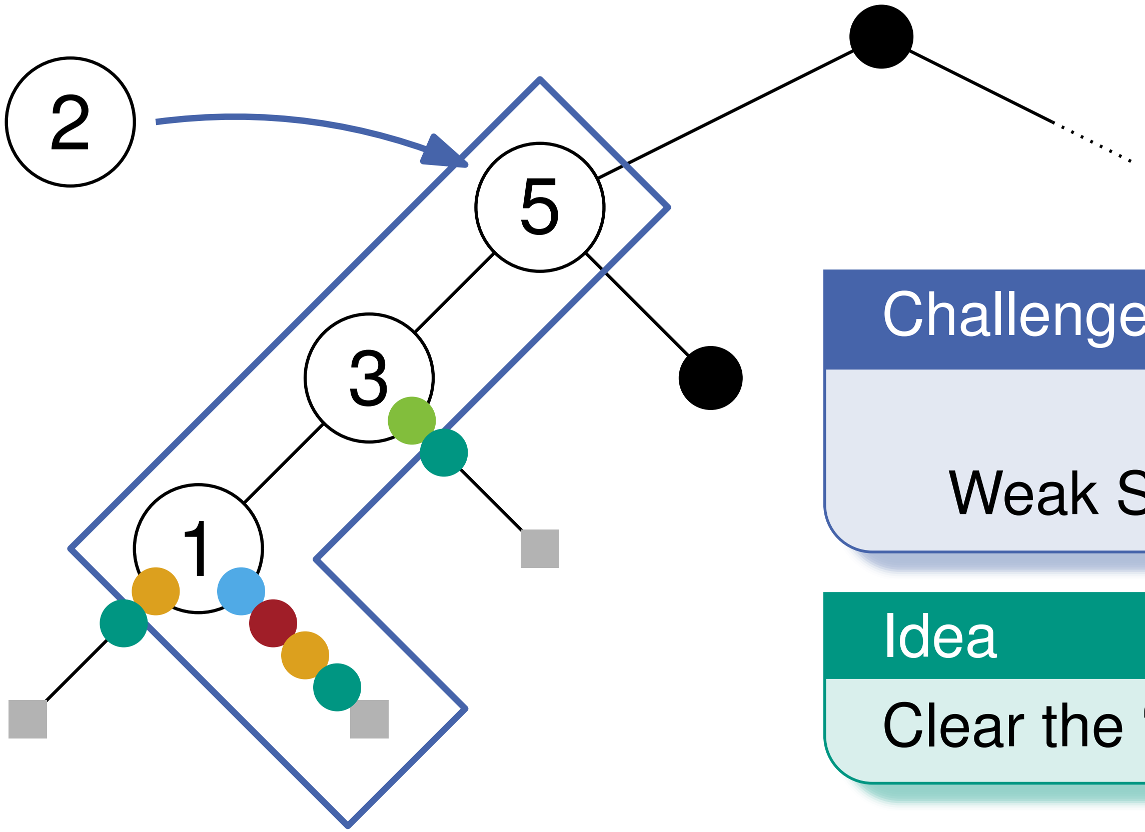

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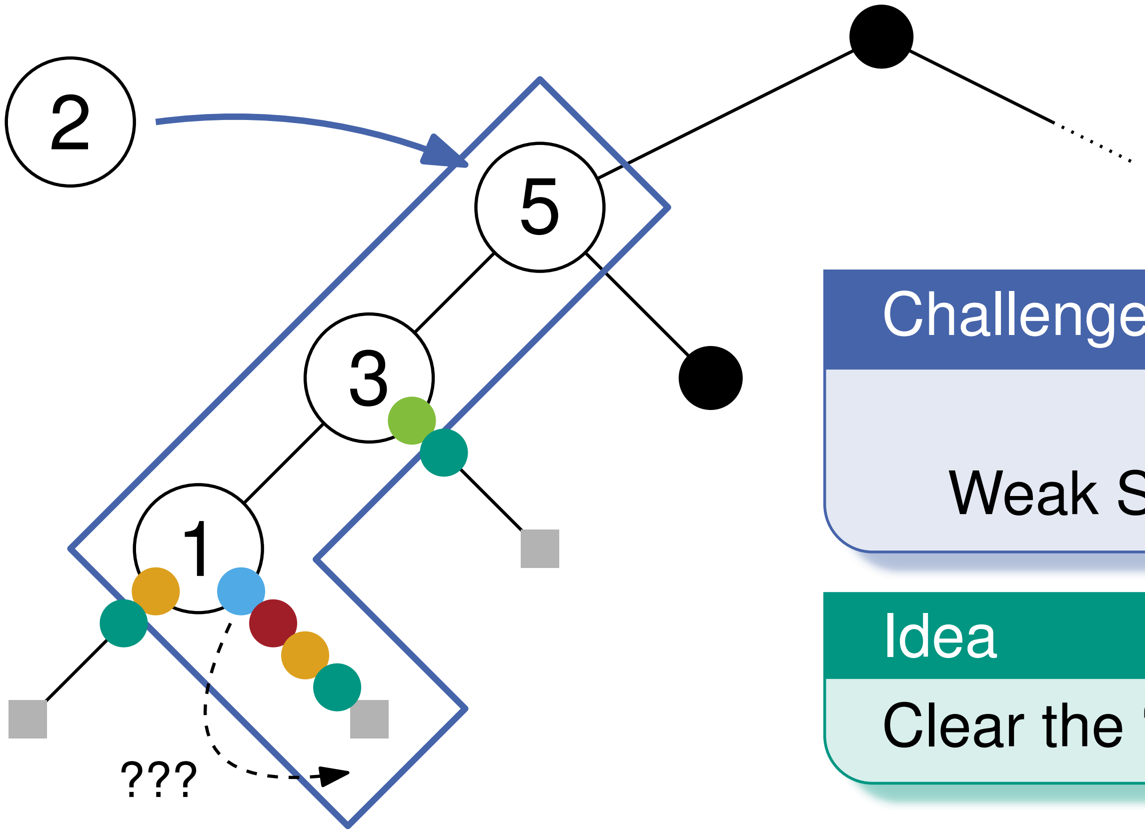

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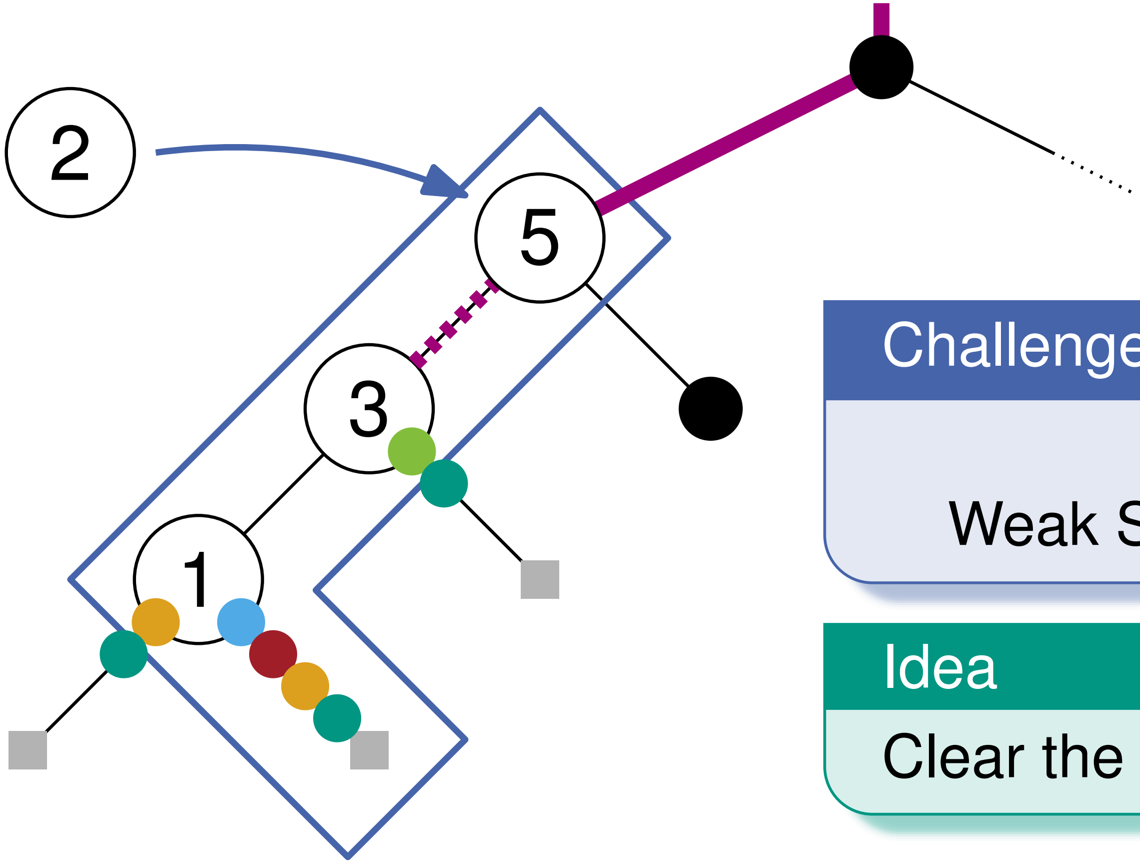

Challenge

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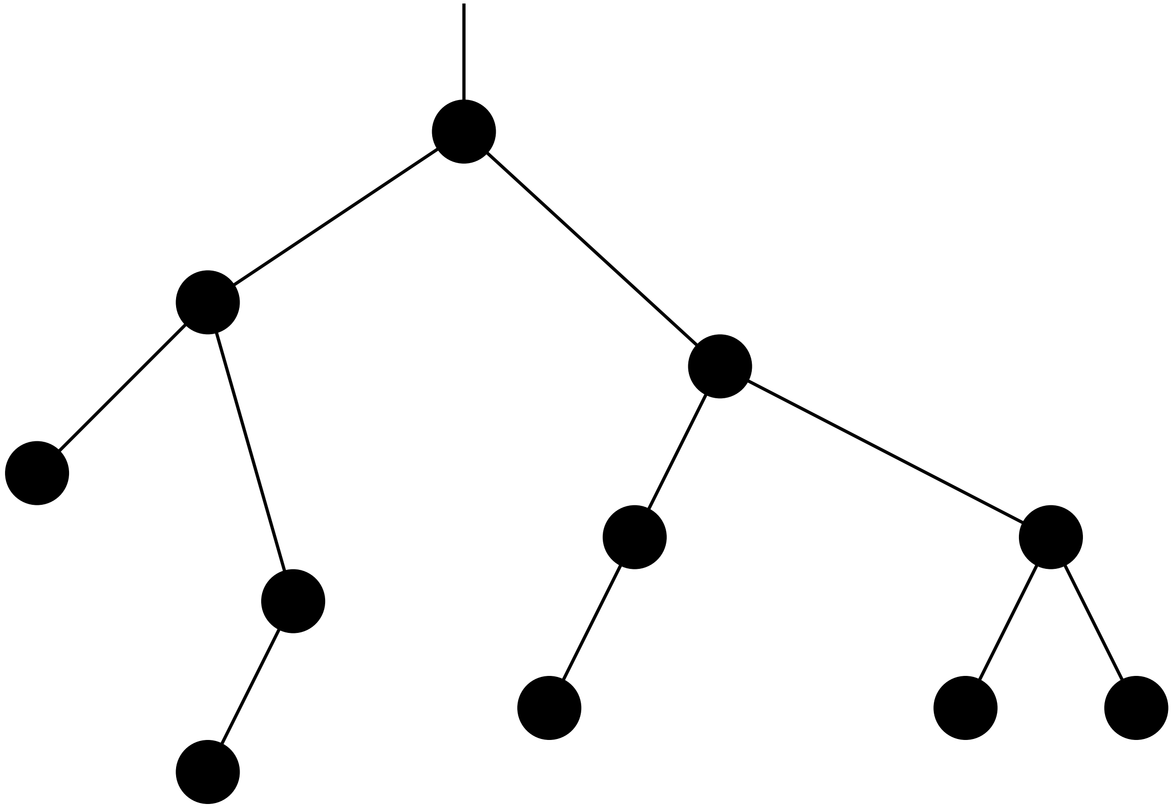
Correctness (Intuition)

Look at search paths along the cleared path.

Zipping Segment Trees - Deletion



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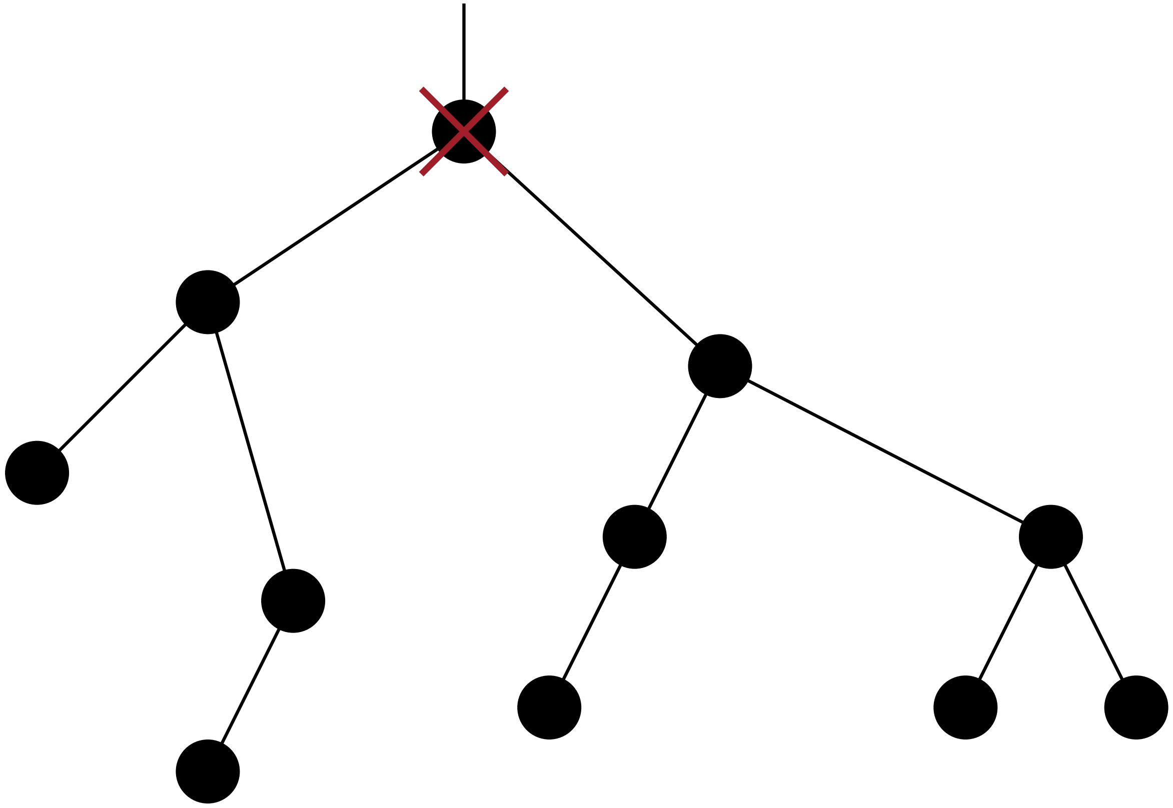

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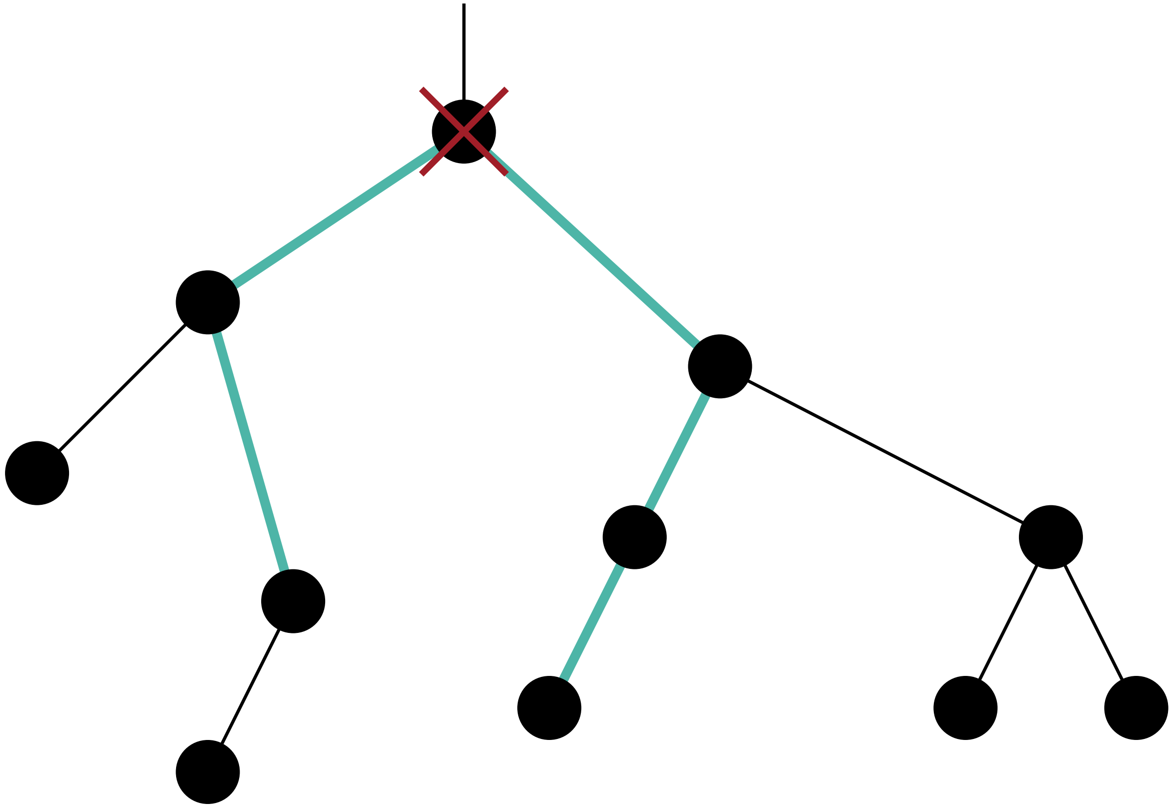

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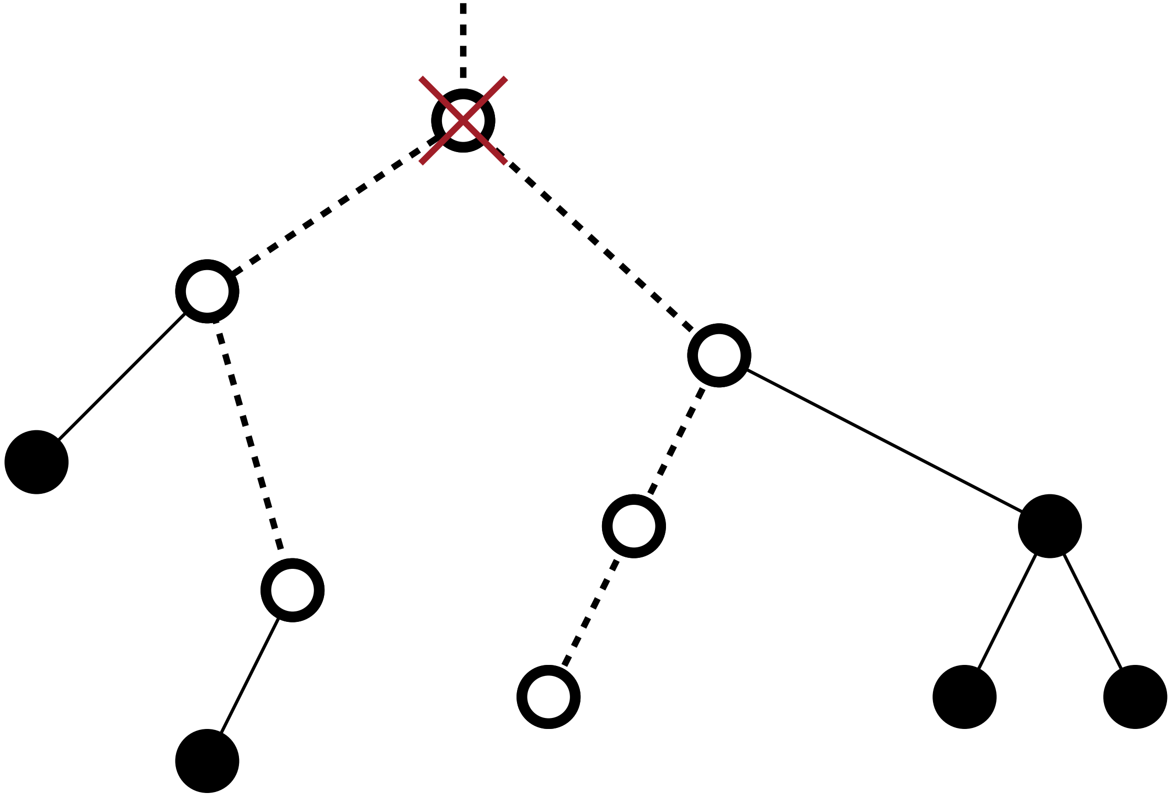

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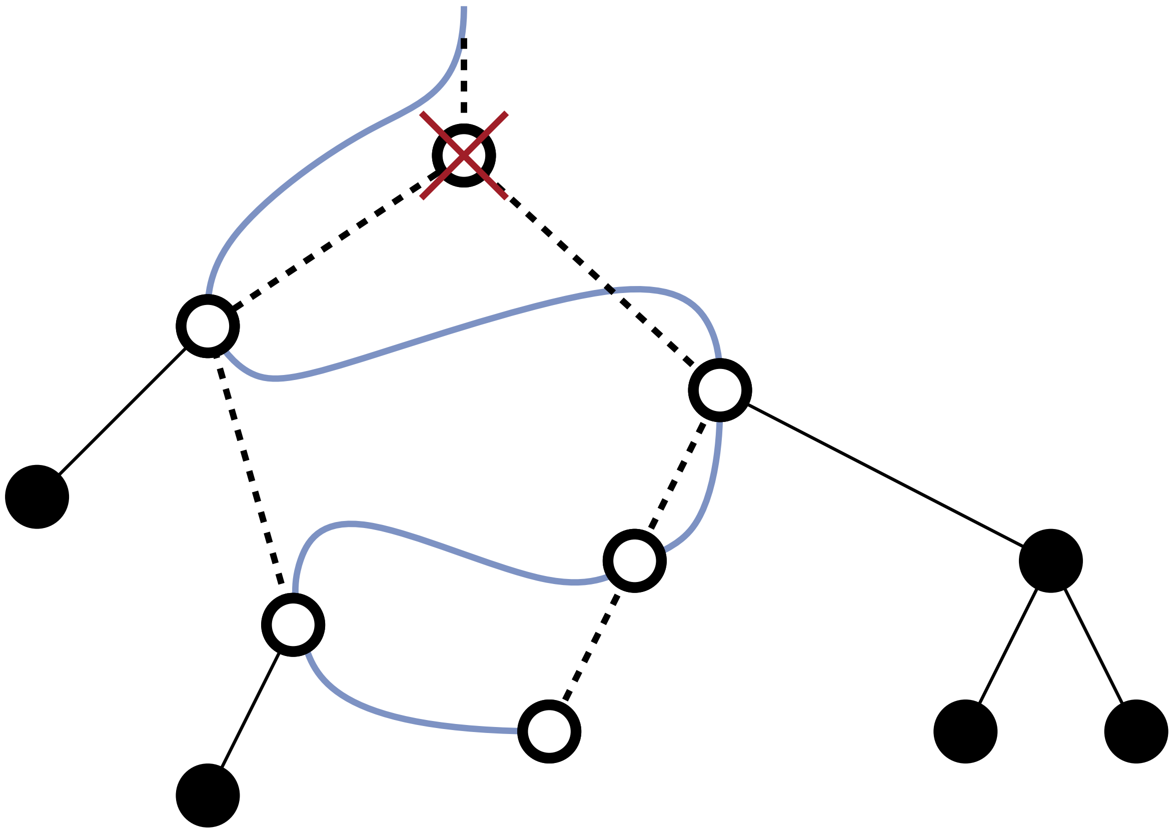

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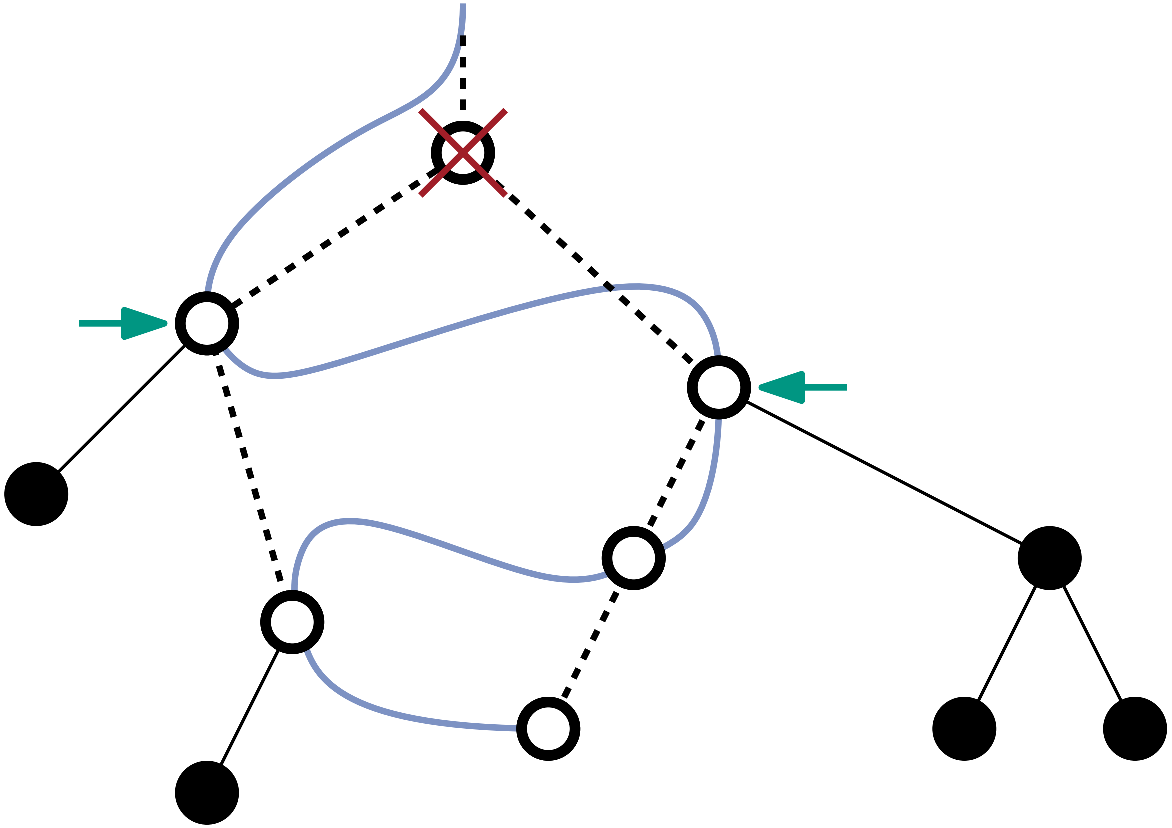

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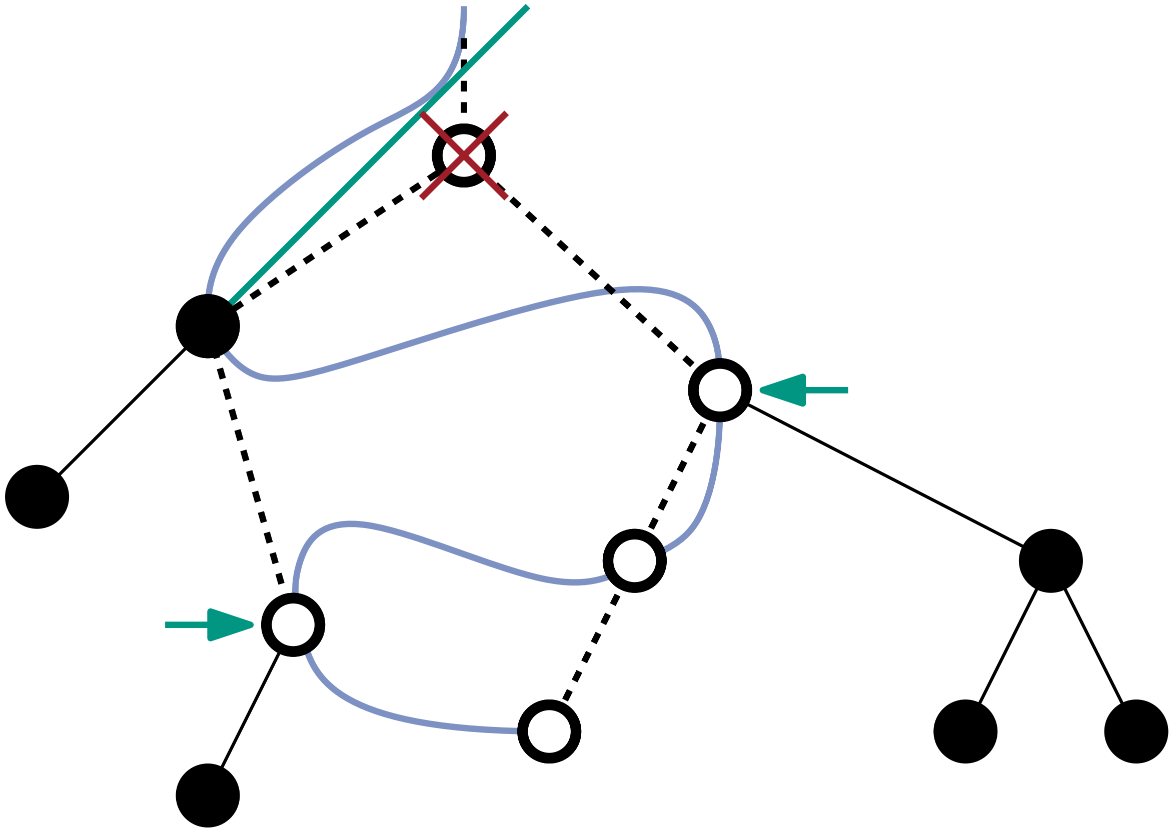

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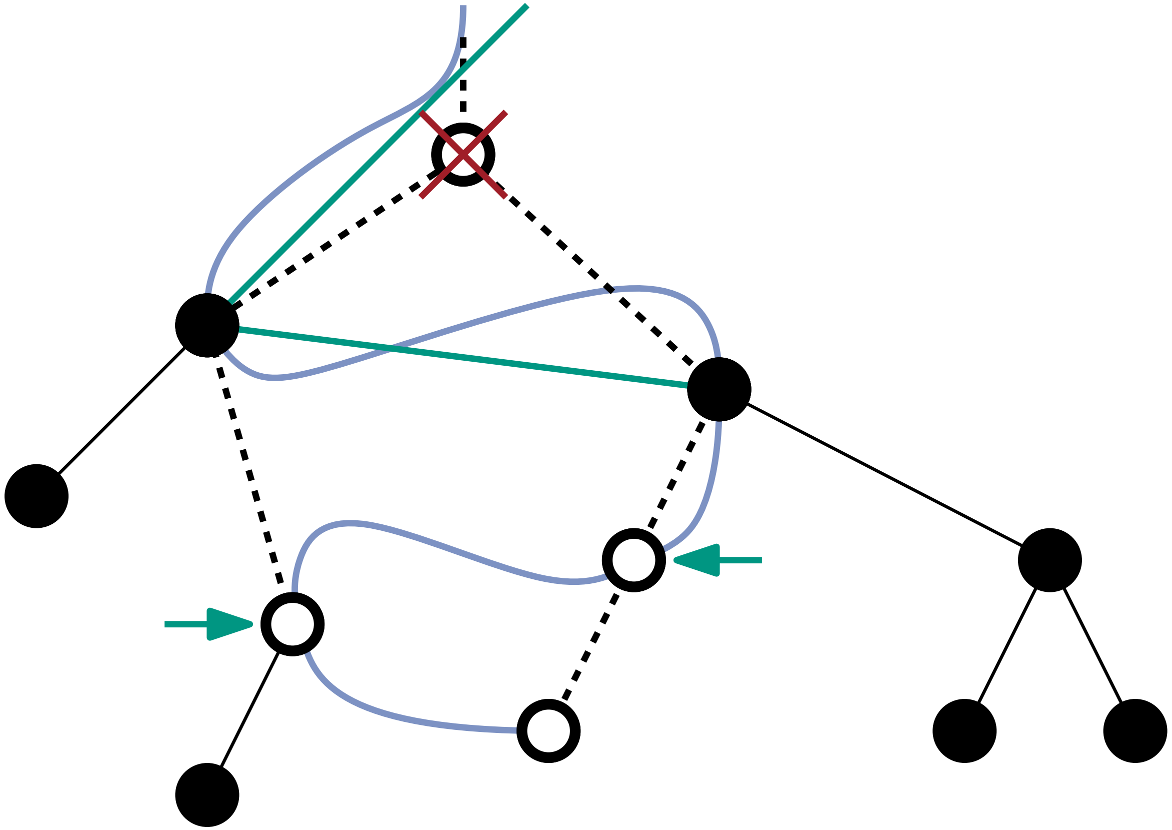

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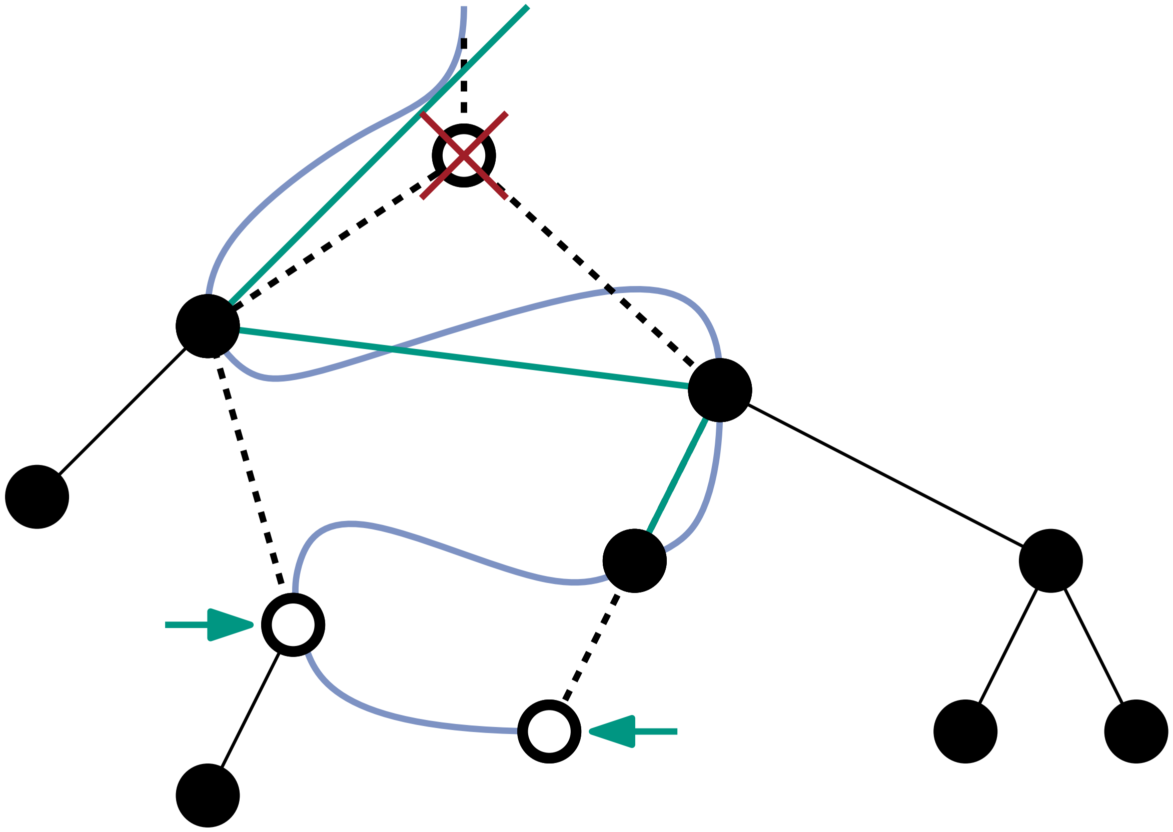

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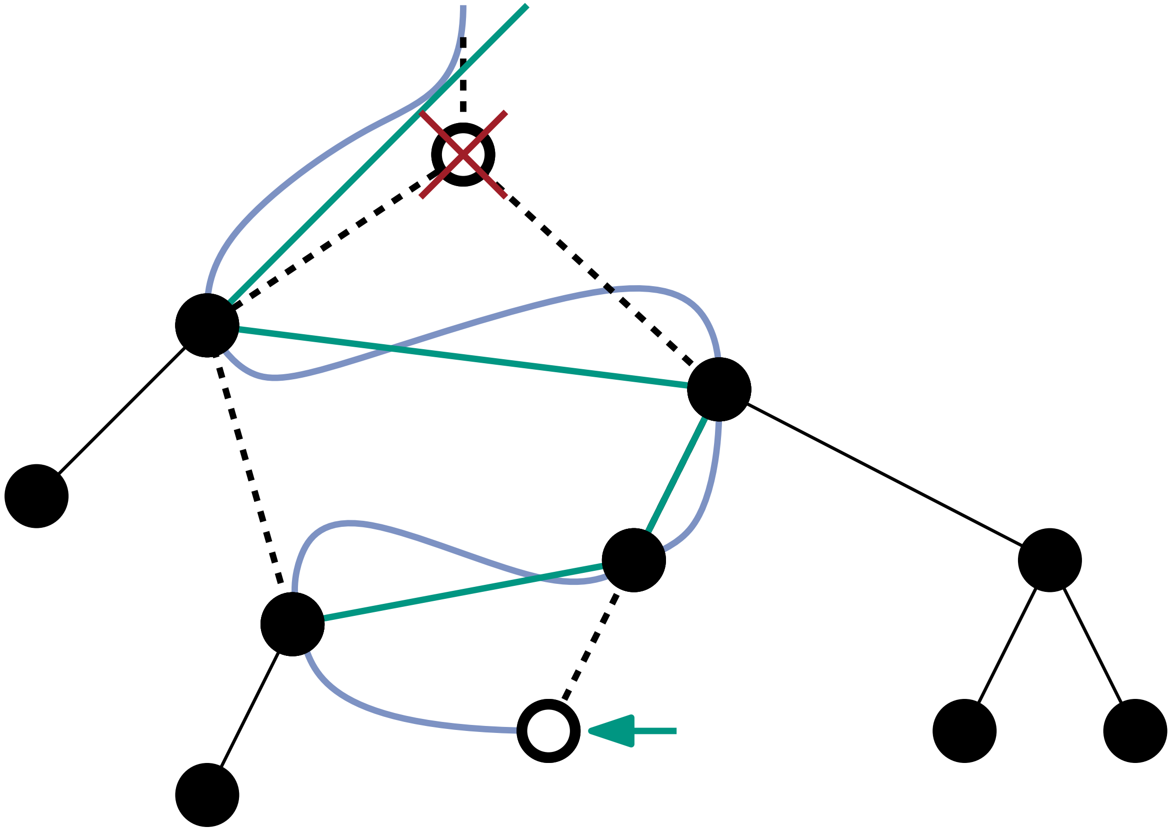

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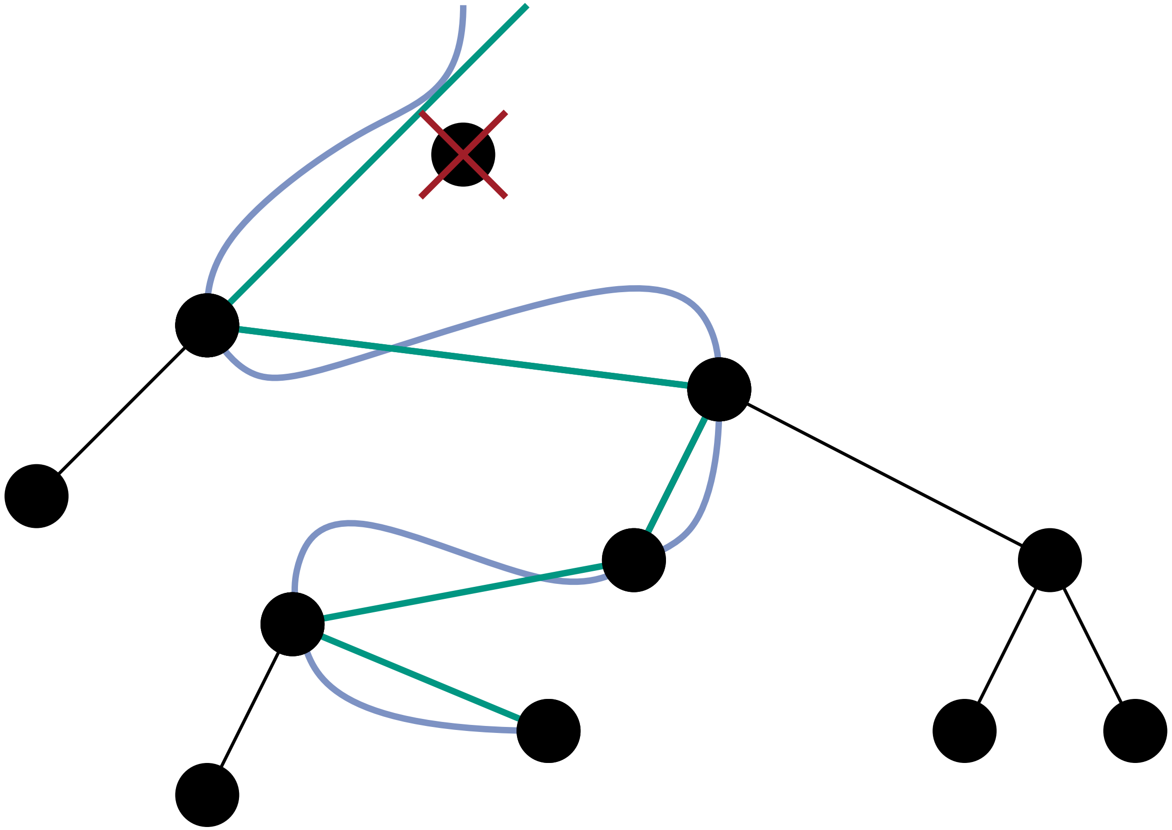

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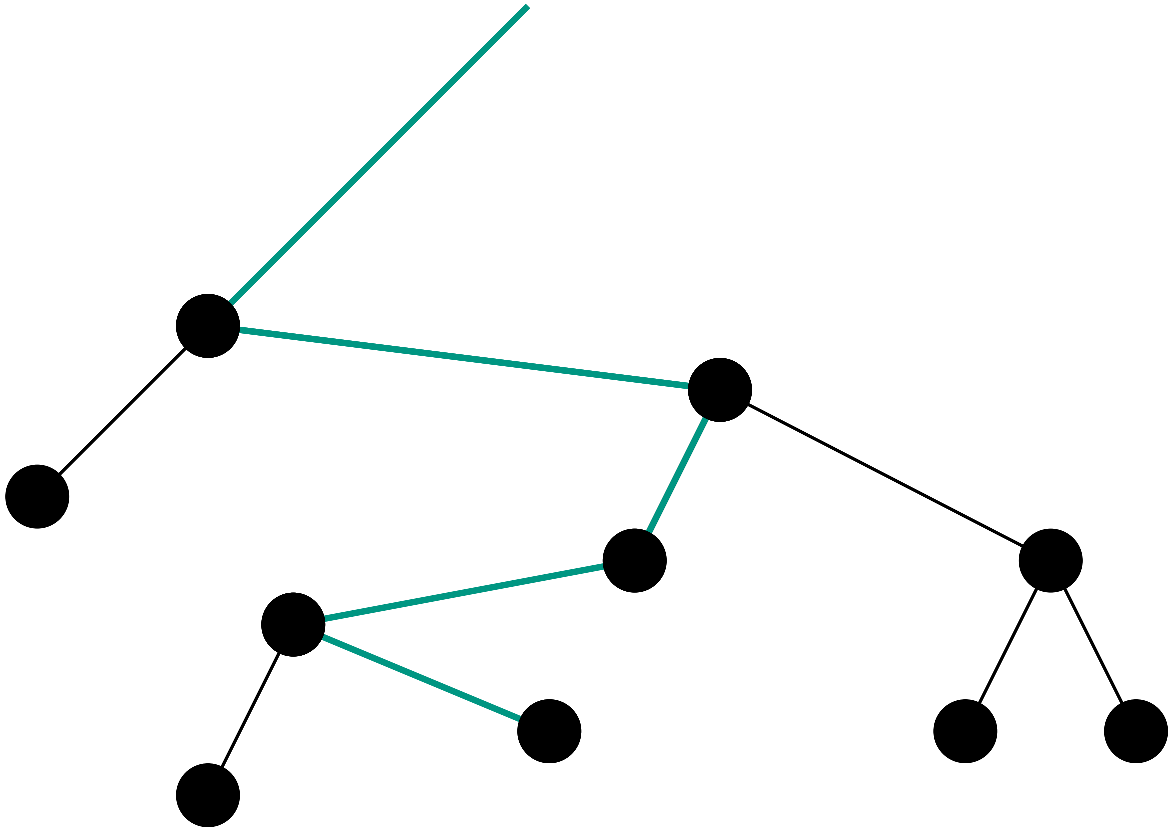

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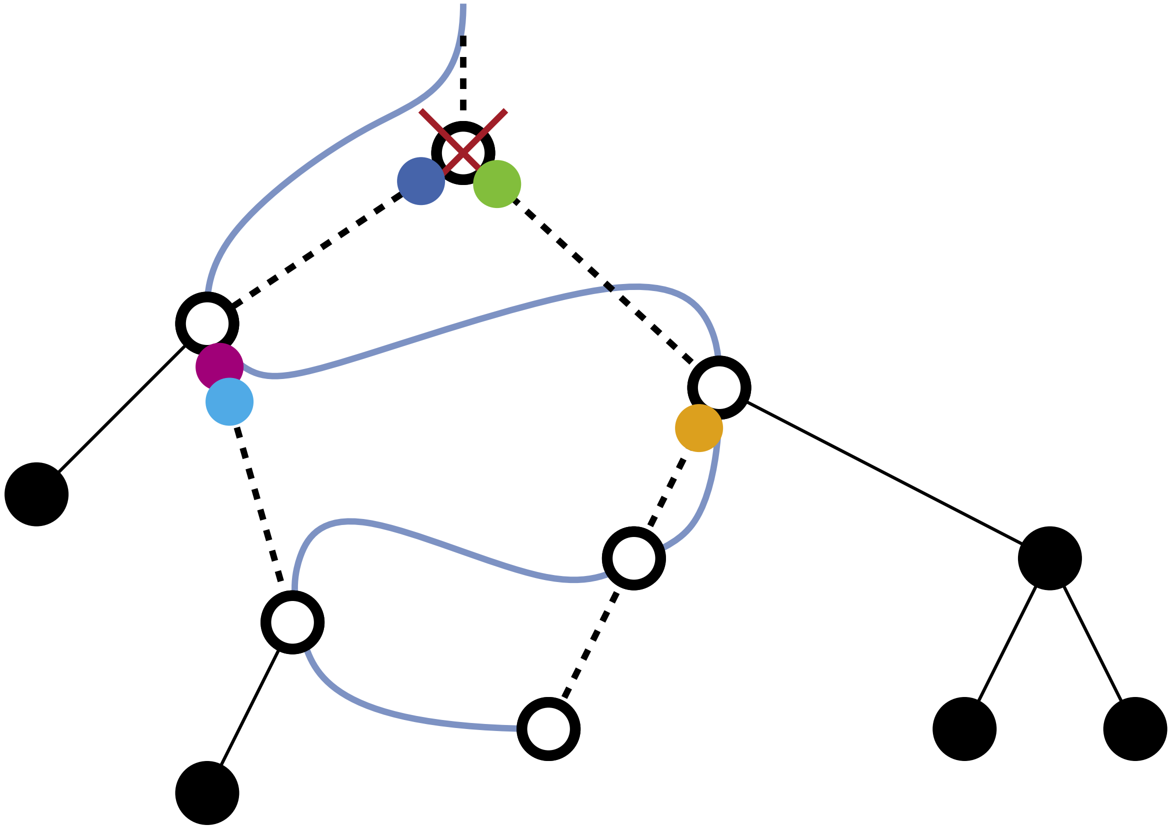

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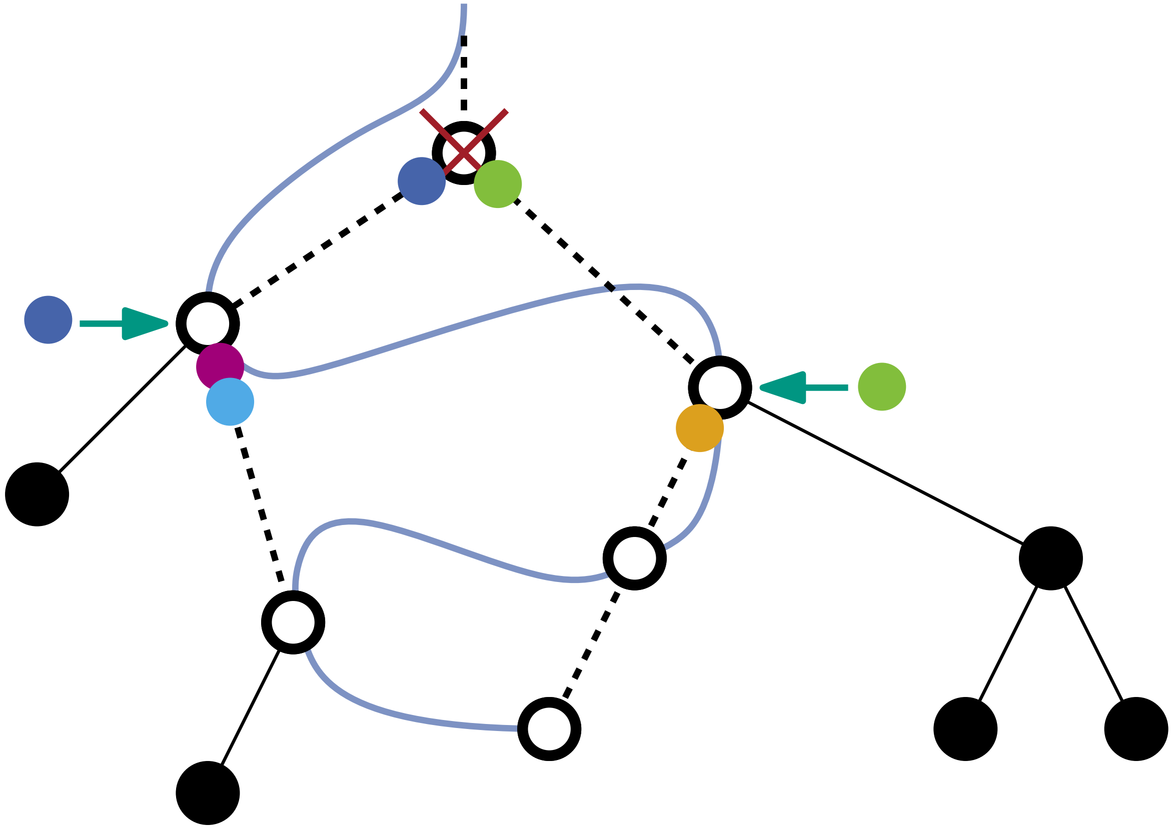

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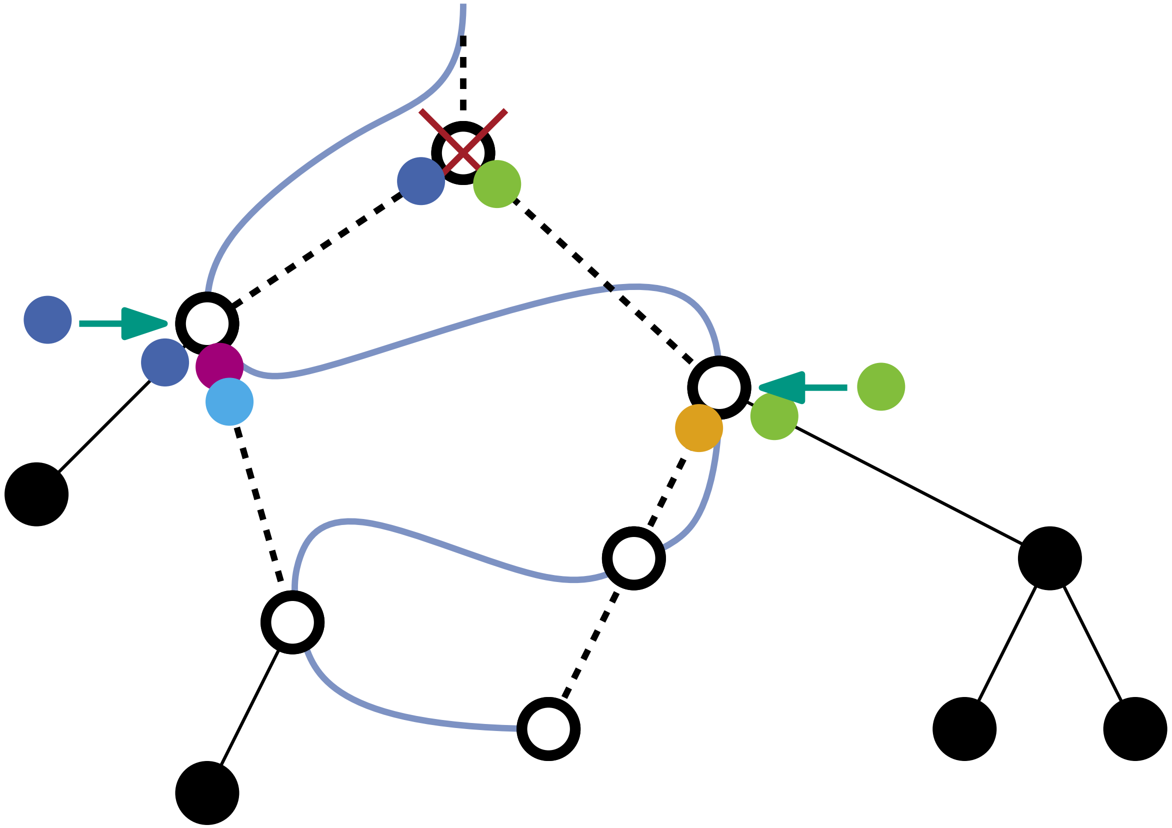

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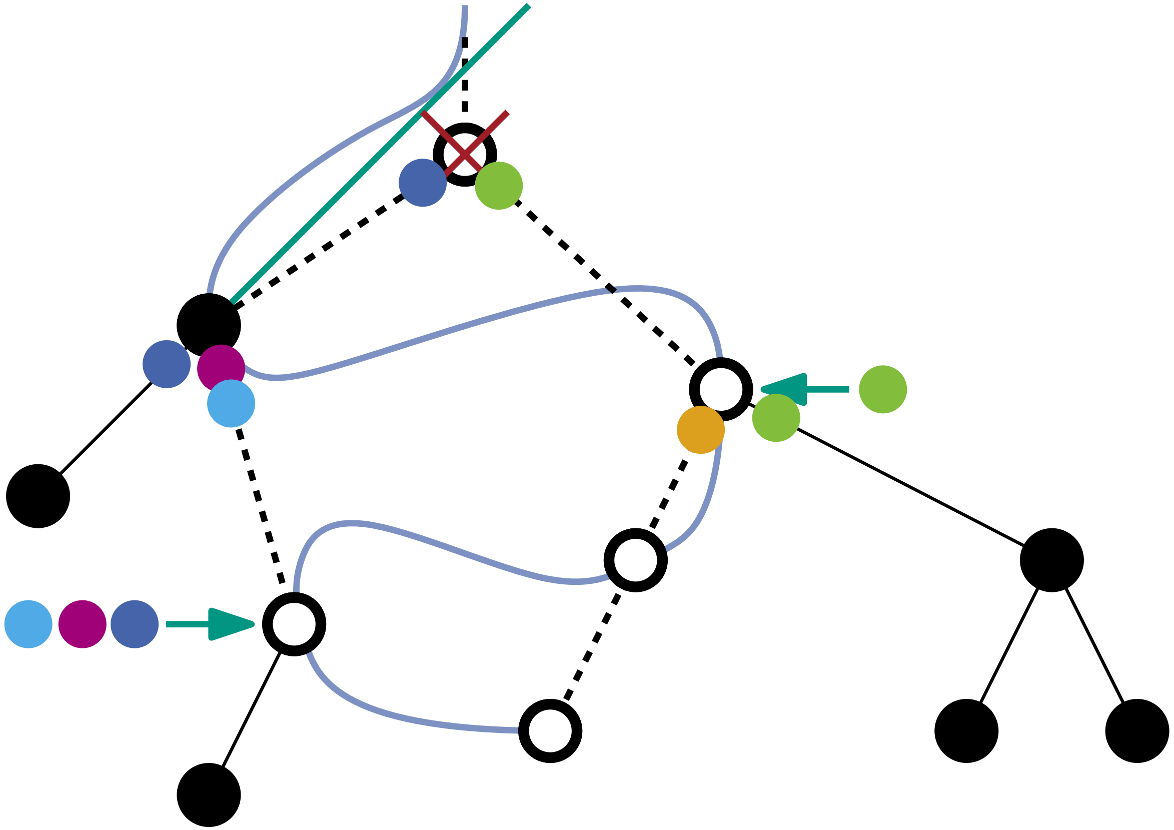

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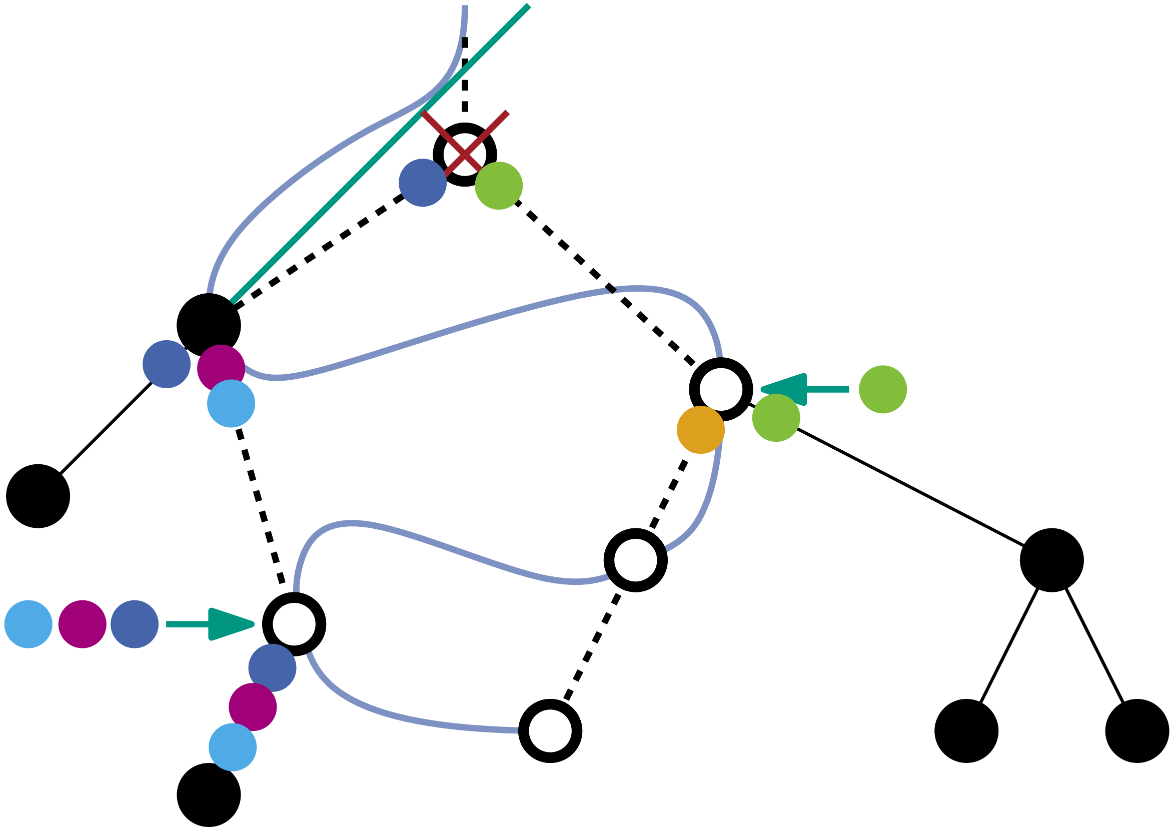

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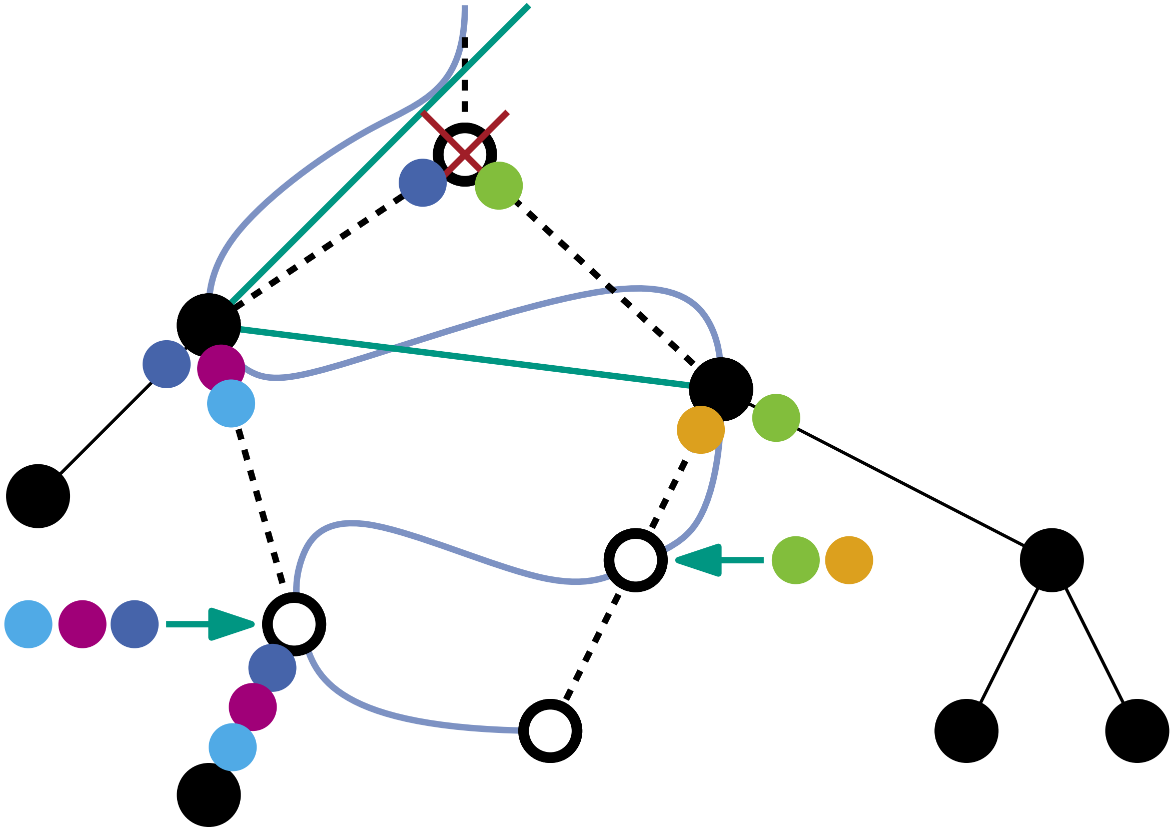

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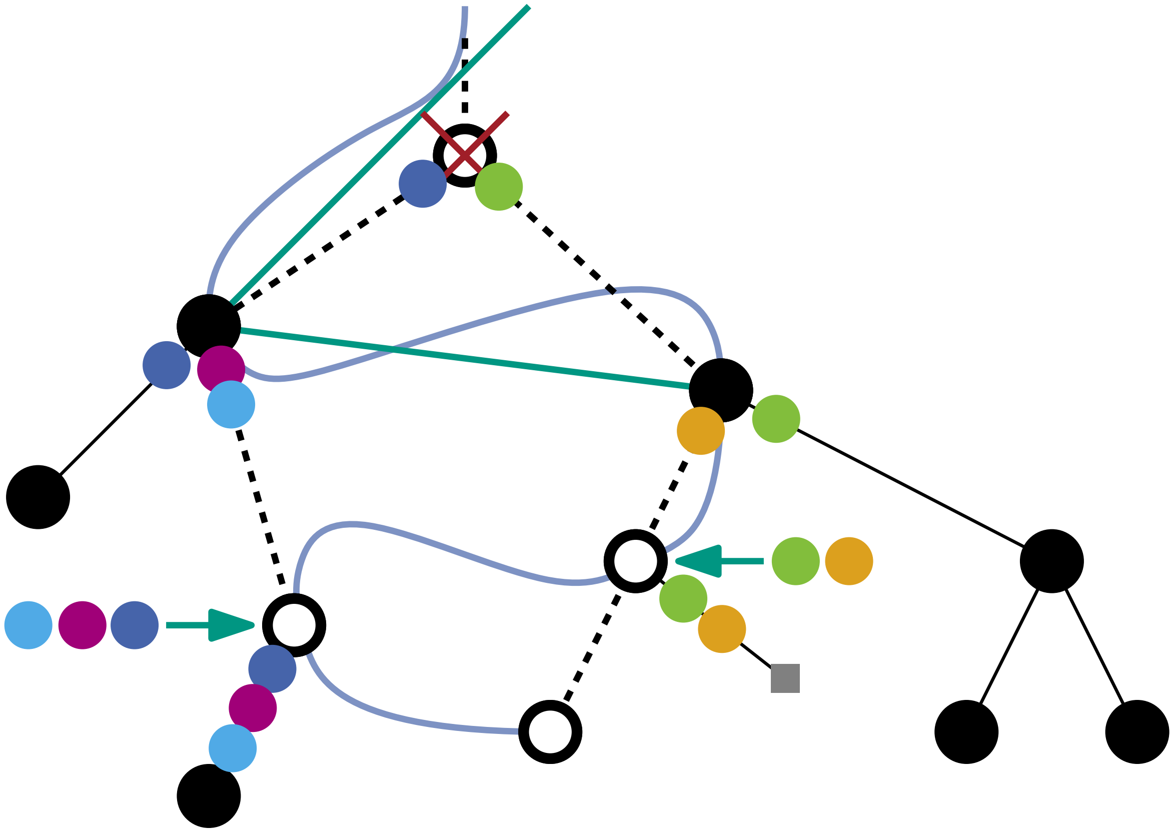

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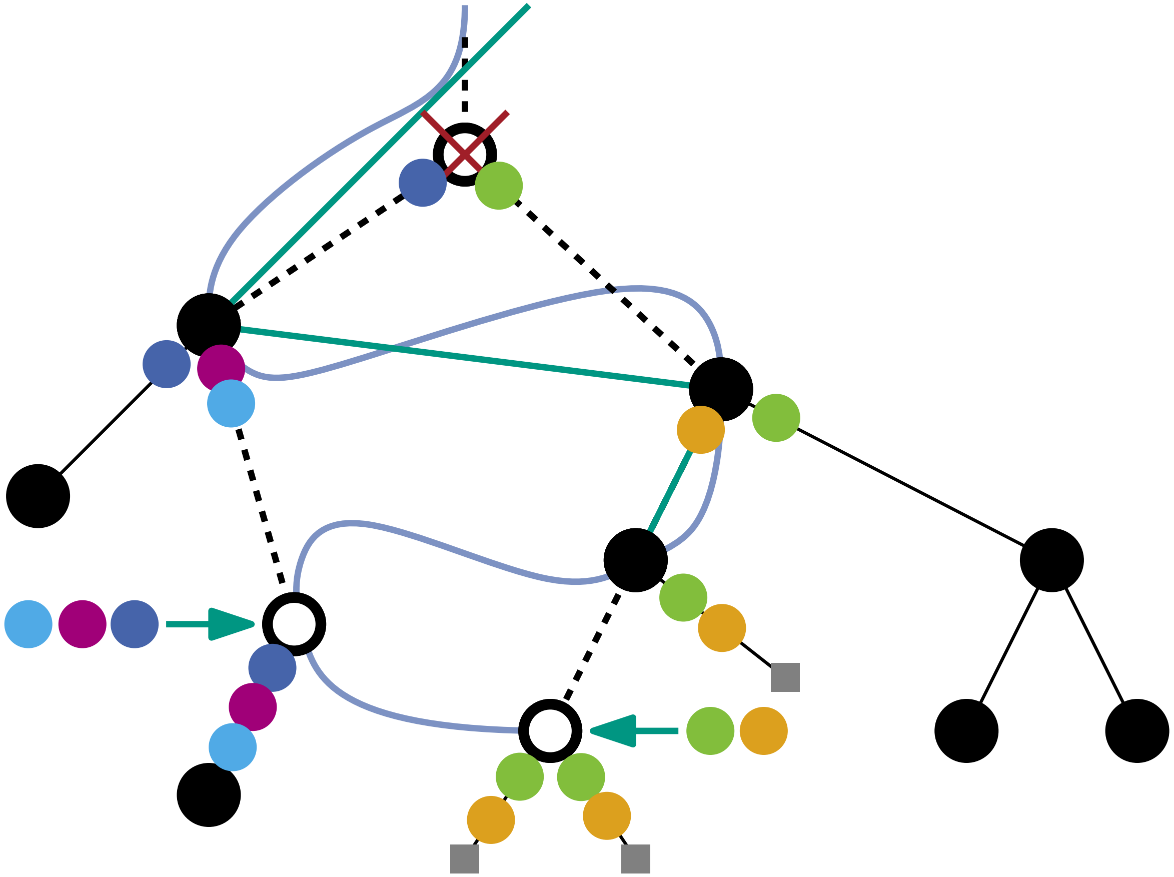

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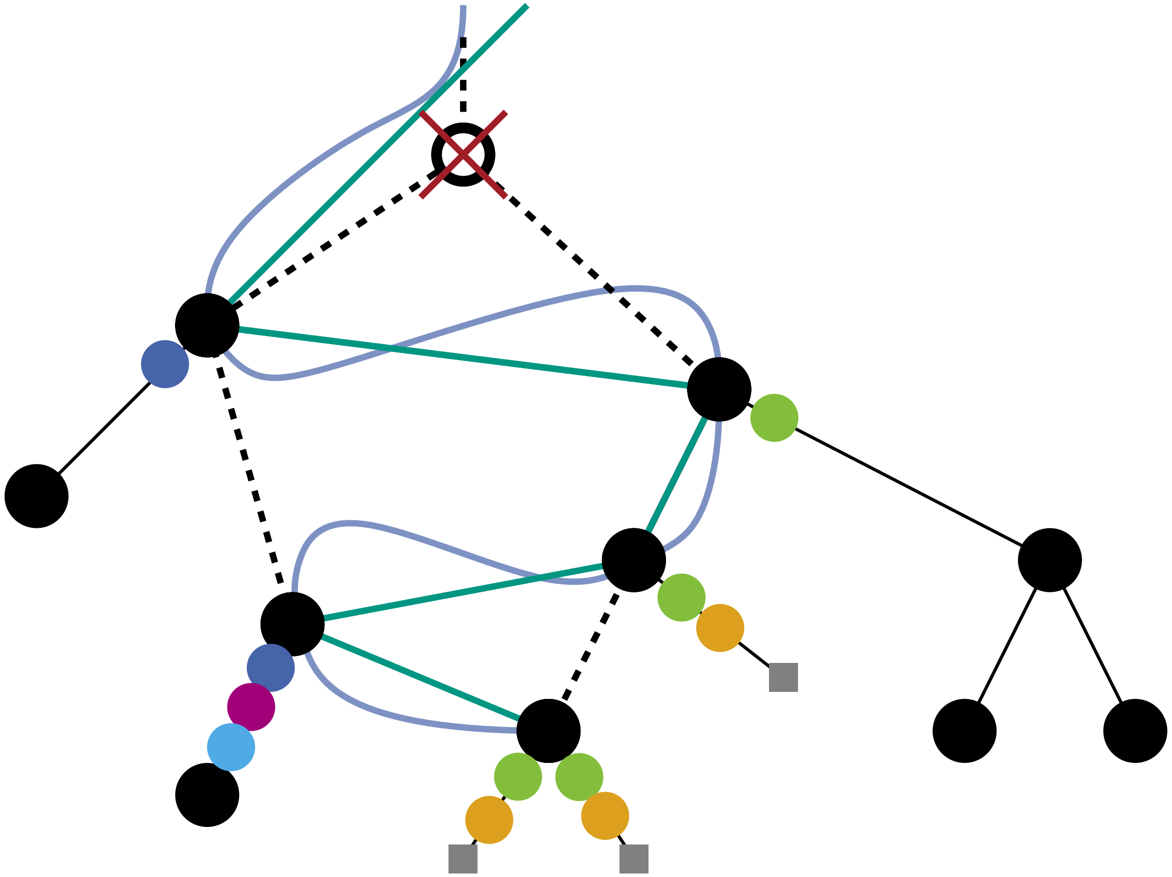
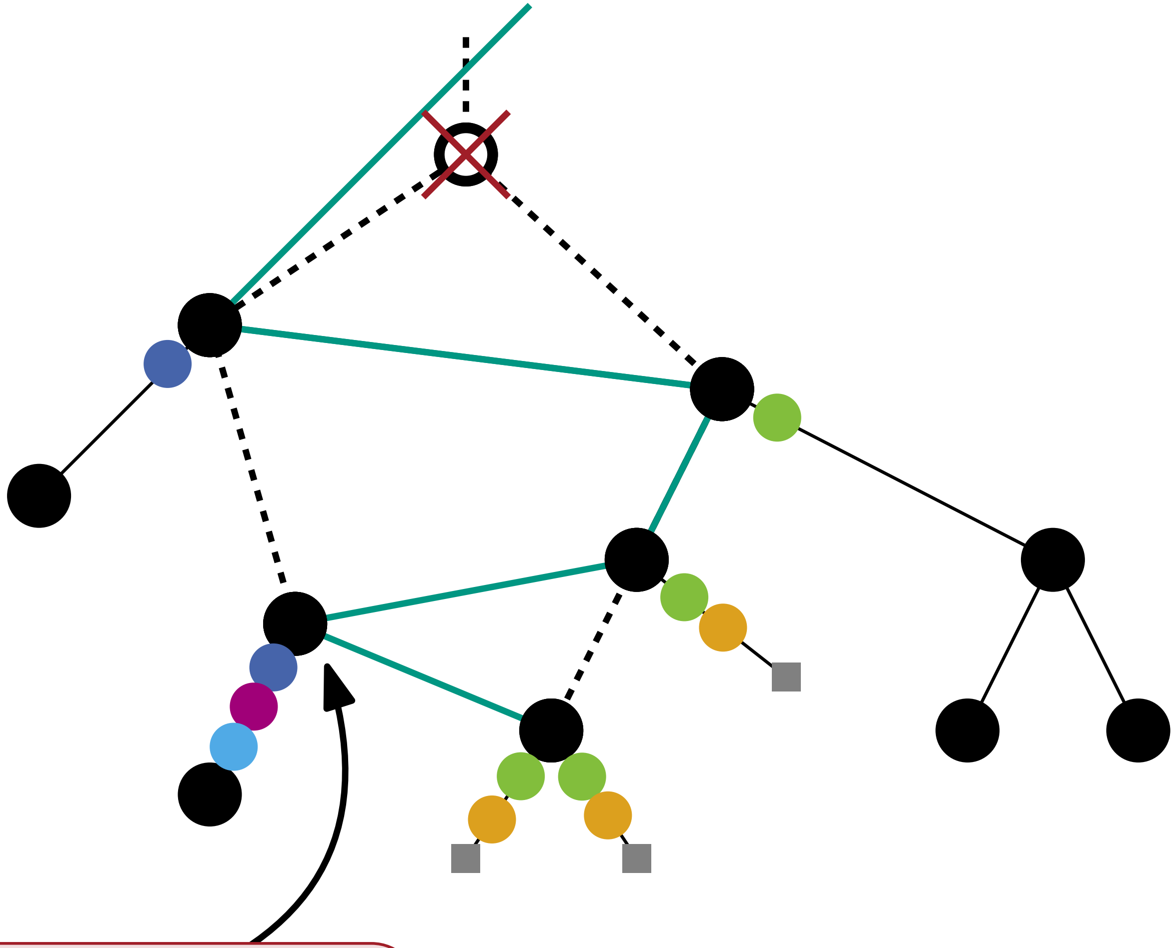


Zipping Segment Trees - Deletion



What about search paths exiting to the right here?

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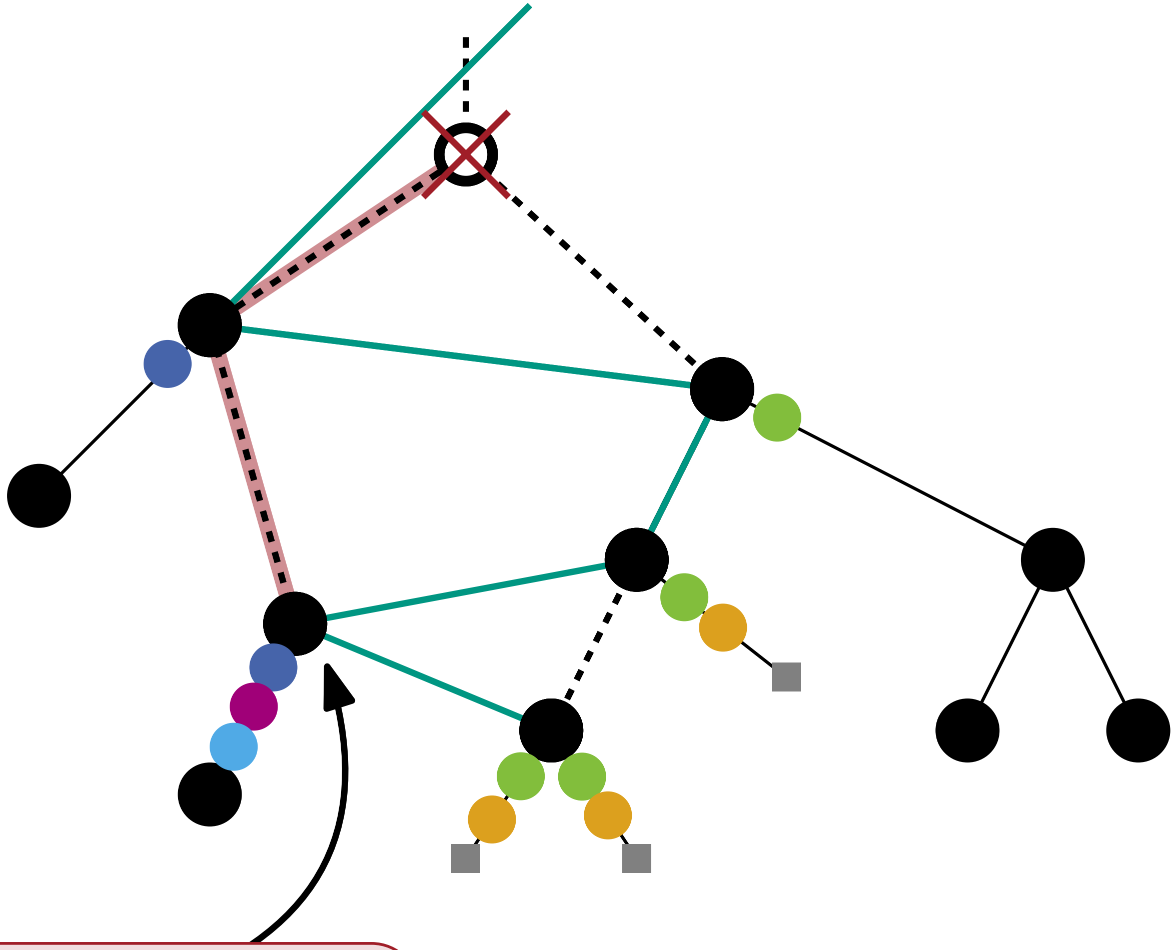

What about search paths exiting to the right here?

Insertion Height

Main Idea

- We want to expect a balanced tree
- Insert node with prob. $\frac{1}{2}$ as leaf, with prob. $\frac{1}{4}$ at height 1, $\ldots$

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Karlsruhe Institute of Technology

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“Random” Variant

- Flip a coin until hitting “heads”

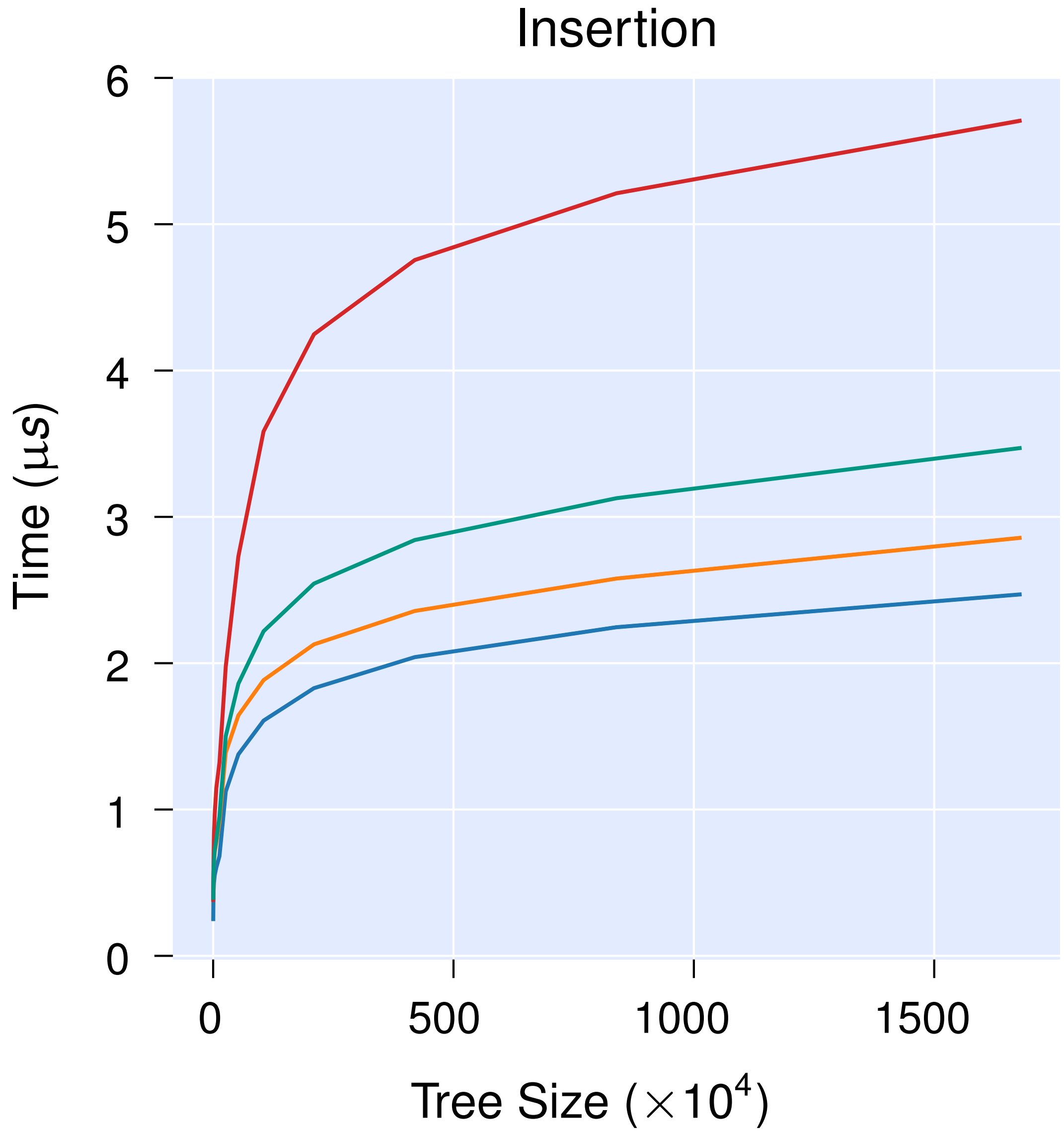
“Hashing” Variant

- Hash the node’s value (or its memory address, or . . .)
- Use the bits as a stream of coin flips
- Advantage: Don’t need to store the rank at the node!

Experimental Results

[B, Wagner. SEA 2020.]

1. Create tree with *n* random intervals (*x* axis)
2. Insert *k* new random intervals
3. *y* axis: Time for step 2 divided by *k*



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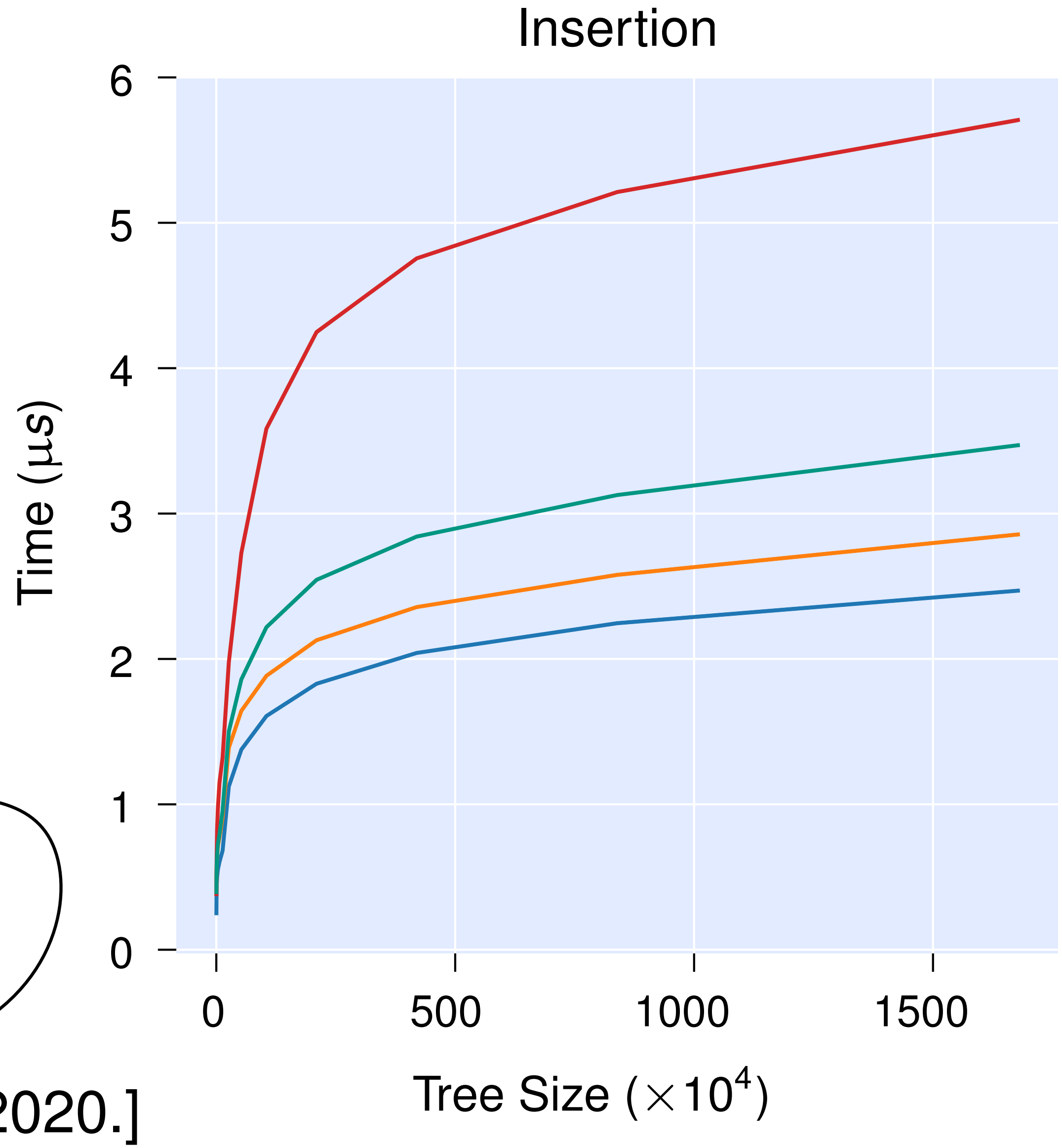

🟦 Red-Black
🟧 Weight-Balanced
🟥 Zip (Hashing)
🟩 Zip (Random)

[B, Wagner. ALENEX 2020.]

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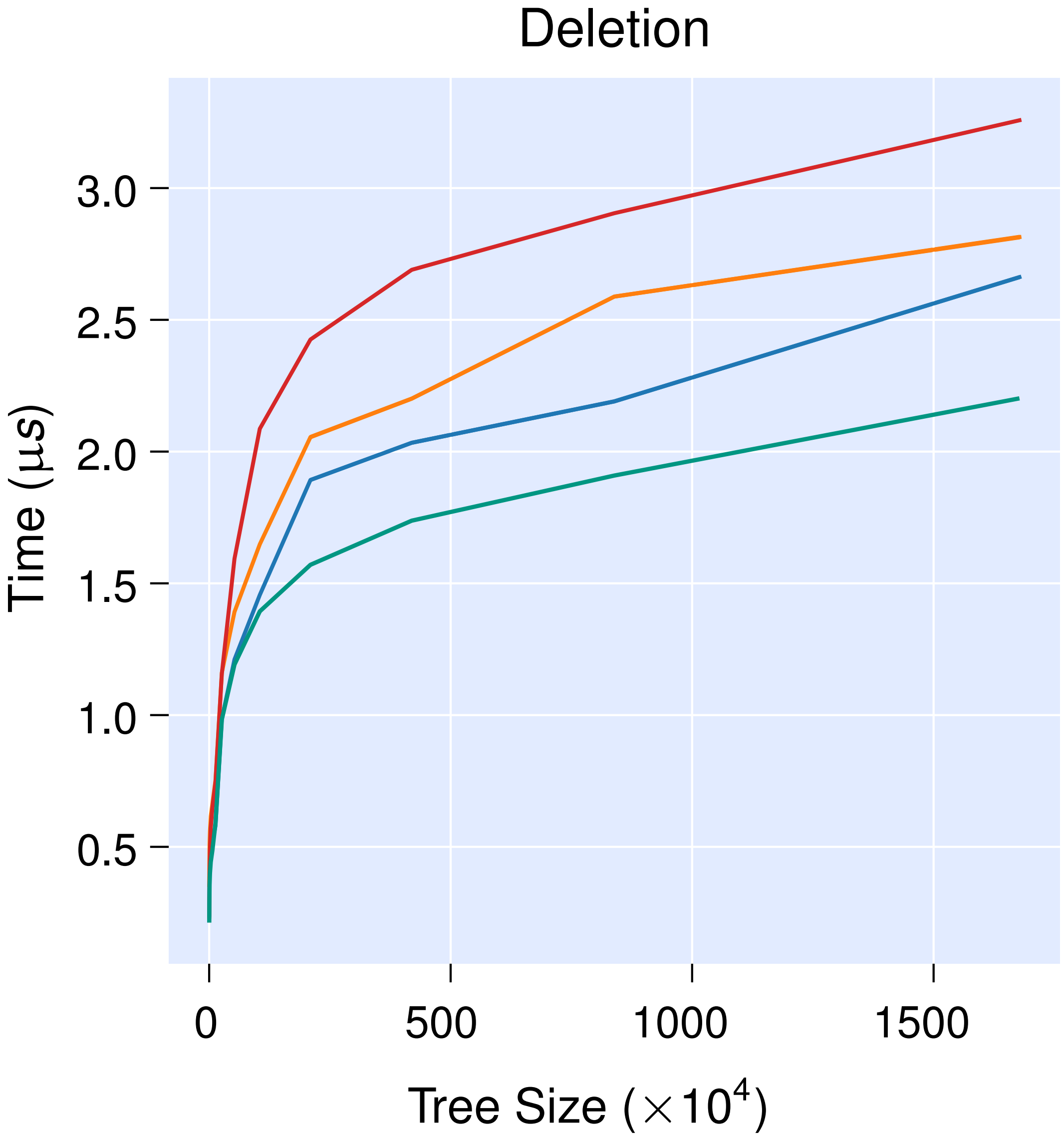

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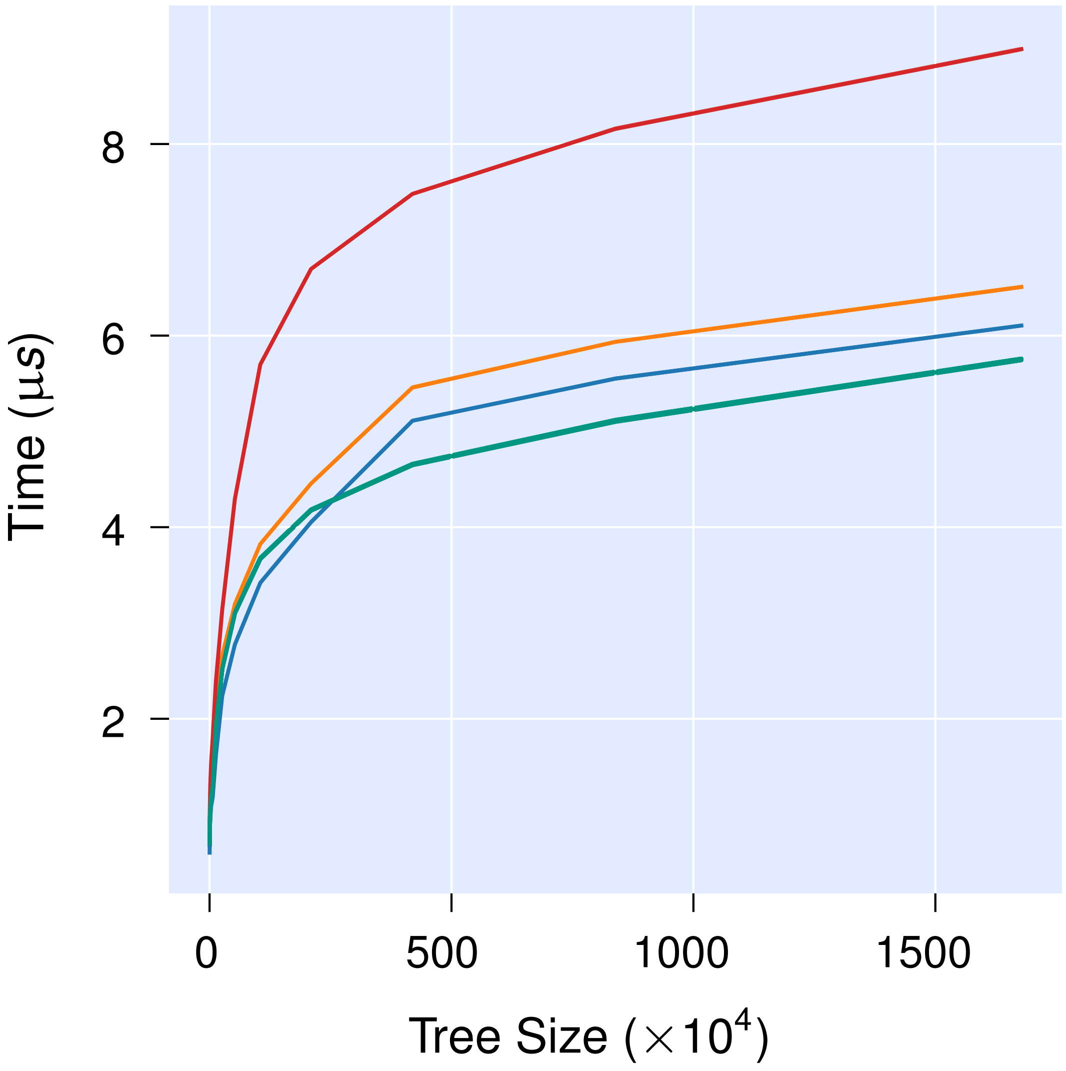
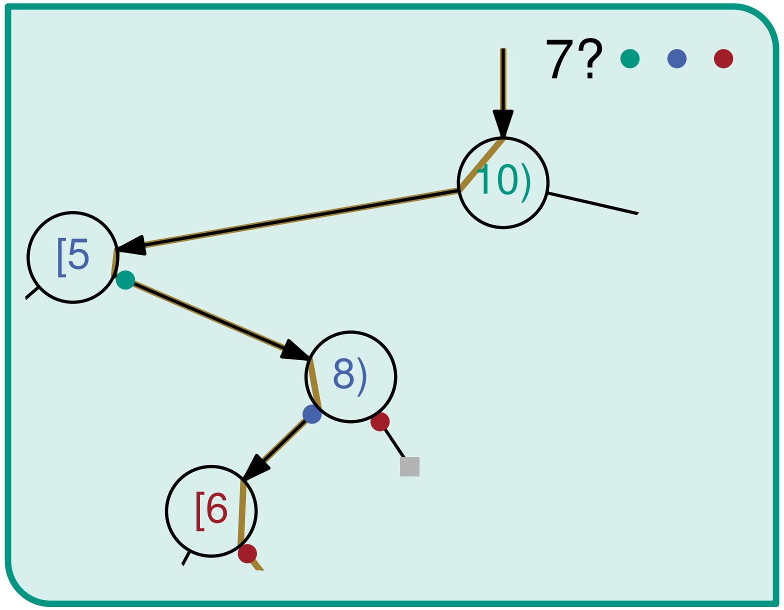
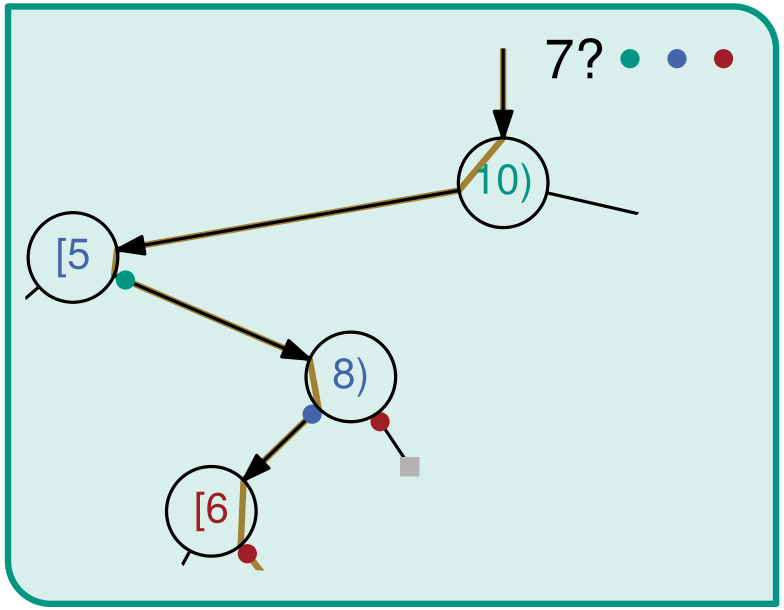
Move

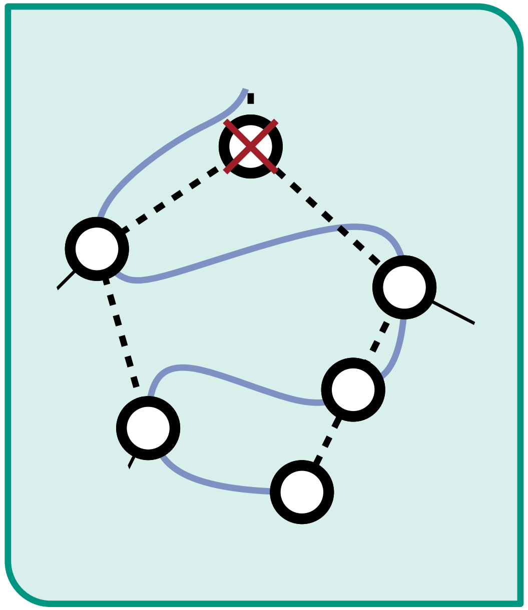


Conclusion



Conclusion





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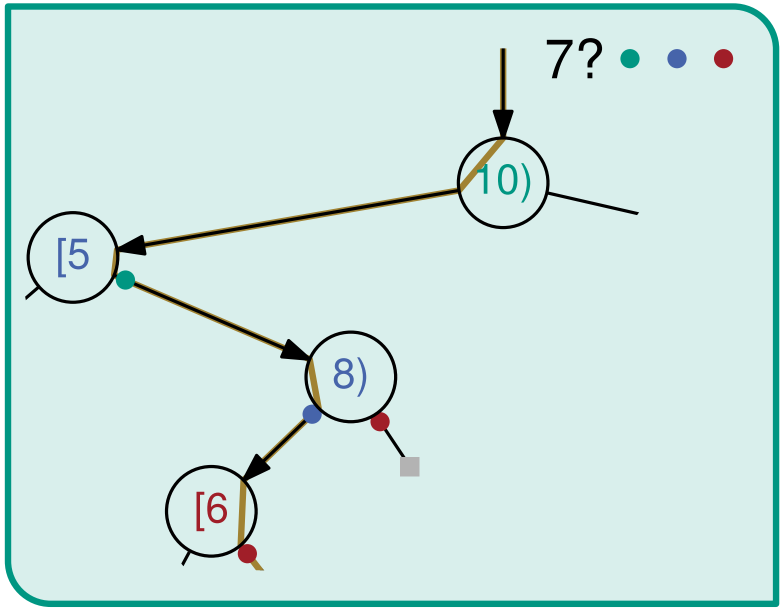
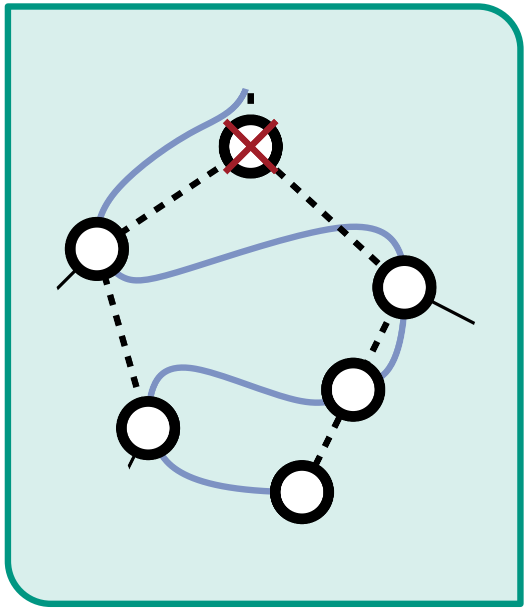
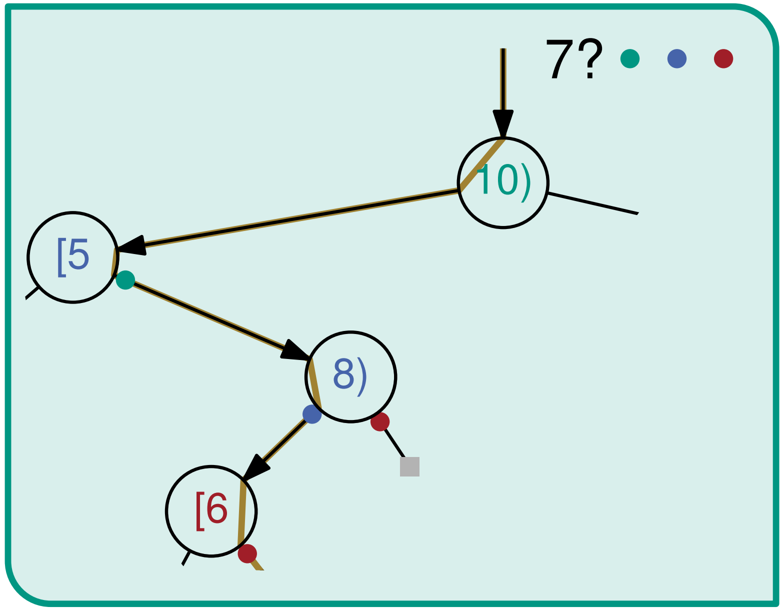
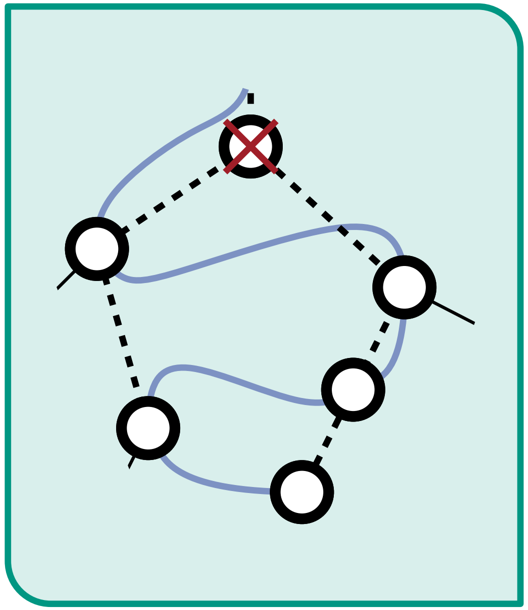




Zipping
Segment
Trees

Conclusion

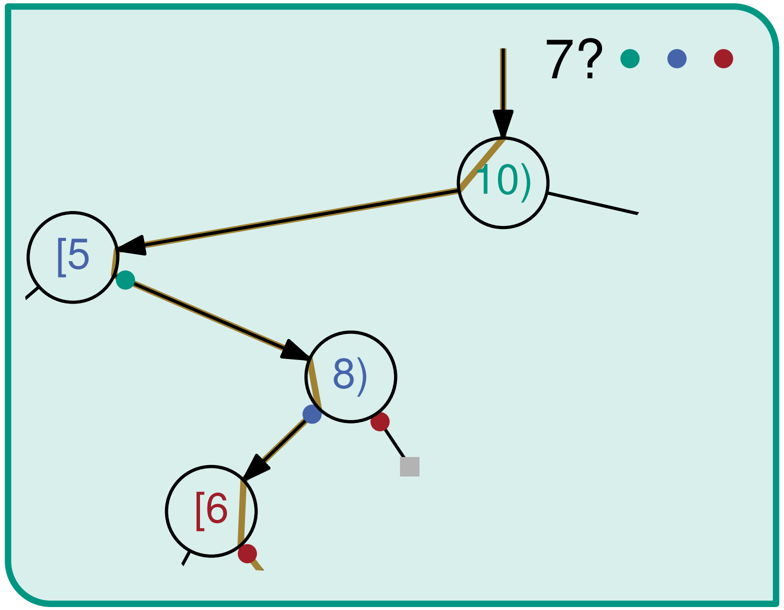


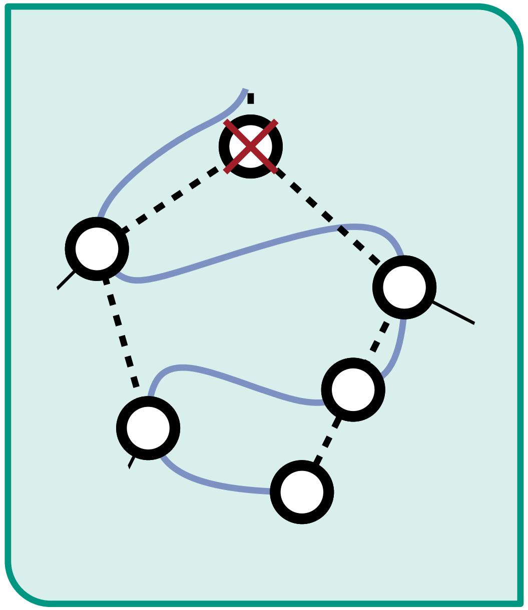


Zipping Segment Trees

- Most efficient choice for deletions and moves

Conclusion





Zipping
Segment
Trees

- Most efficient choice for deletions and moves
- Next step: Tuning Zip Trees!