Storing Set Families More Compactly with Top ZDDs

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Abstract

- Purpose
  - Compress
    zero-suppressed binary decision diagram (ZDD)
    ≒ labeled binary directed acyclic graph (DAG)
- Method
  - Expand a tree compression algorithm to DAGs
- Result
  - Theoretic: Exponentially smaller than input
  - Experimental: Smaller than a related research
    in almost all cases

Contents

- Preliminary
  - ZDD
  - Tree compression algorithms
- Proposed data structure
  - Construction algorithm
  - Complexity analysis
- Experiment
- Conclusion

Preliminary

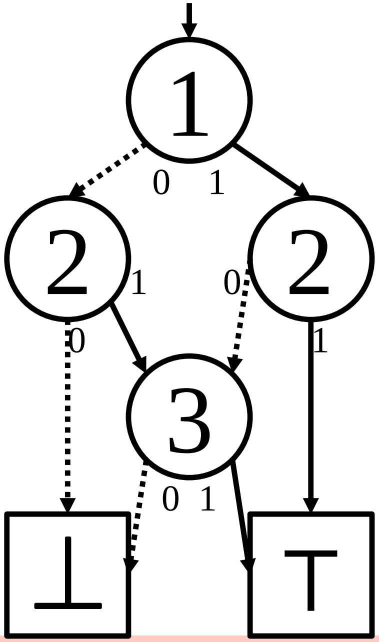
ZDD
DAG compression
Top tree compression

ZDD

- Zero-suppressed binary decision diagram [Minato 93]
  - Labeled binary directed acyclic graph
  - Represents a family of sets
  - Share equivalent subgraphs

- Terminology
  - Branching nodes
    - Label
    - 0-edges and 1-edges
  - Sink nodes
    - Top or bottom

{ {1, 2}, {1, 3}, {2, 3} }



Tree compression methods

- Tree grammar
  - Based on grammar compression for strings [Charikar et al. 05]
  - Traversing on compressed representations require linear time to the size of grammar [Busatto et al. 04,], [Lohrey et al. 13]

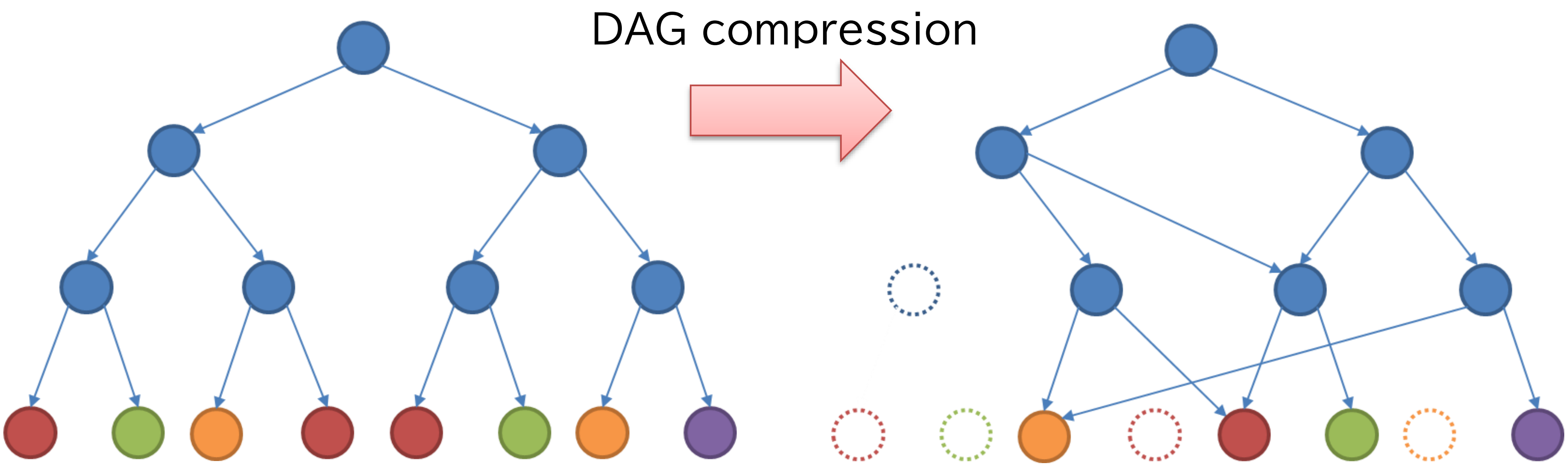
- Succinct data structures
  - Labeled tree: LOUDS [Jacobson 89]
    BP [Munro, Raman 01]
  - Unlabeled tree: [Ferragina et al. 09]

Tree compression

- Transform-based compression
  - Shares equivalent sub structures
  - DAG compression [Downey et al. 80]
    - Shares all equivalent subtrees
  - Top DAG compression [Bille et al. 13]
    - Shares equivalent subcomponents

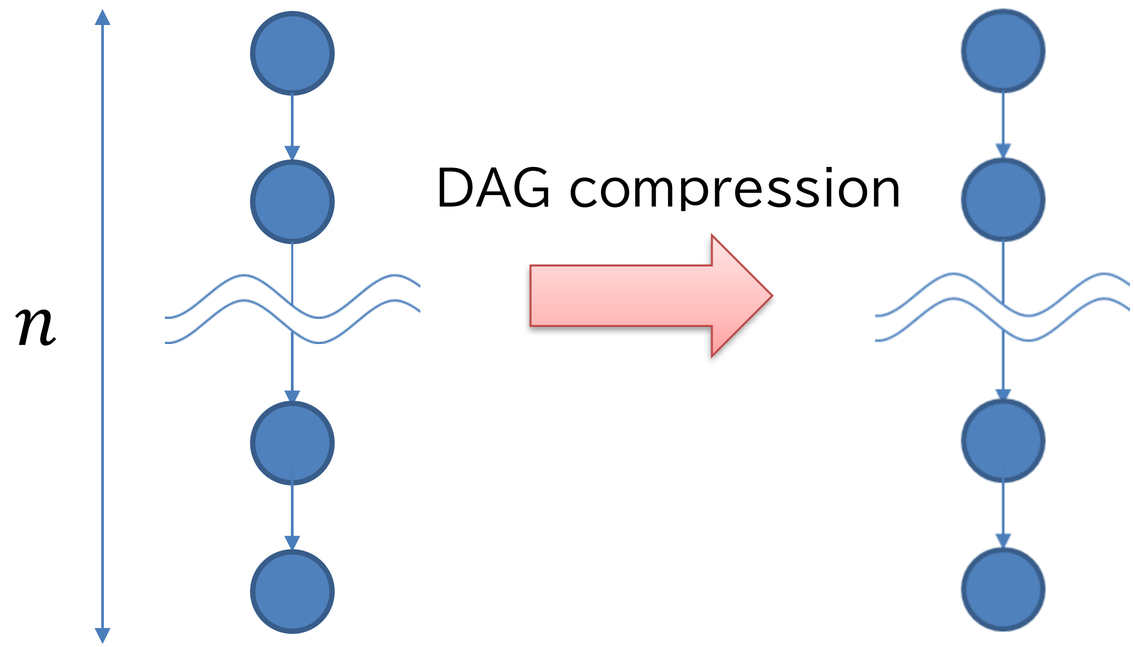
DAG compression

- Compress labeled DAGs
  - [Downey et al. 80]
  - Share all equivalent subtrees



Problem of DAG comp.

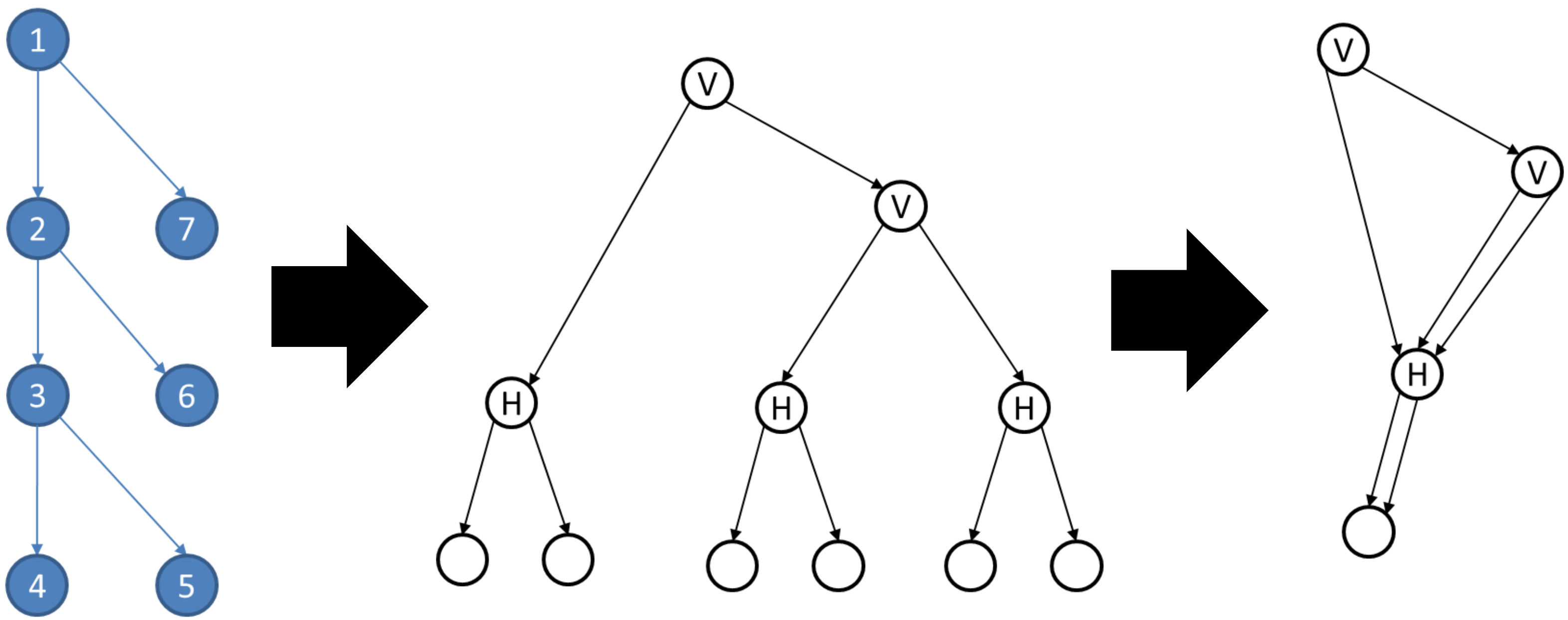
- Cannot compress substructures that repeats vertically
- Example:



Simple, but not compressed

Top DAG compression

- Compress labeled DAGs [Bille et al. 13]
  - Transform an input tree to top tree, and compress the top tree by DAG compression



Input tree T

top tree $\mathcal{T}$

top DAG $\mathcal{TD}$

Top DAG compression

- In comparison to DAG compression:
  - [Best case] $O(n / \log_\sigma n)$ times smaller
  - [Worst case] $O(\log_\sigma n)$ times larger
- Greedy construction [Bille et al. 13]
  - #node = $O(n \log \log_\sigma n / \log_\sigma n)$
  - Proof is in [Hübchle-Schneider and Raman 15]
- Optimal construction [Lohrey et al. 17], [Dudek, Gawrychowski 18]
  - #node = $O(n/\log_\sigma n)$ (information theoretic lowerbound)

(n: #node of the input tree, σ: #label)

Top tree

- A binary tree $\mathcal{T}$ that represents the way to decompose the input tree T
  - Each node of the top tree corresponds to a cluster of T
  - The root of the top tree corresponds to whole T
  - A cluster is an induced subgraph of a set of connected edges
  - Every cluster has at most 2 boundary nodes
  - A cluster is made by horizontal or vertical merge of 2 clusters that have the same node as a boundary node

Top tree

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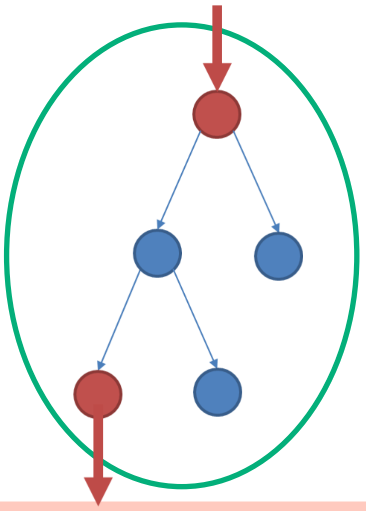
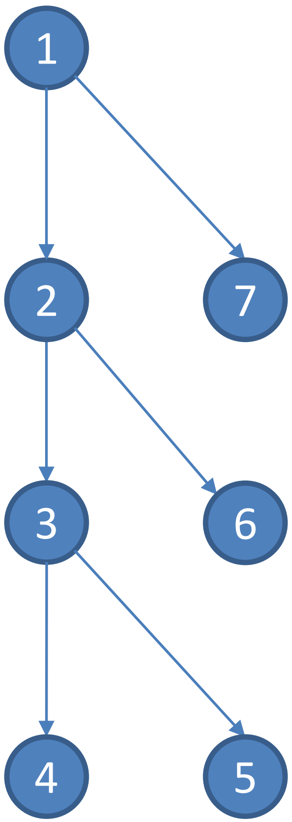
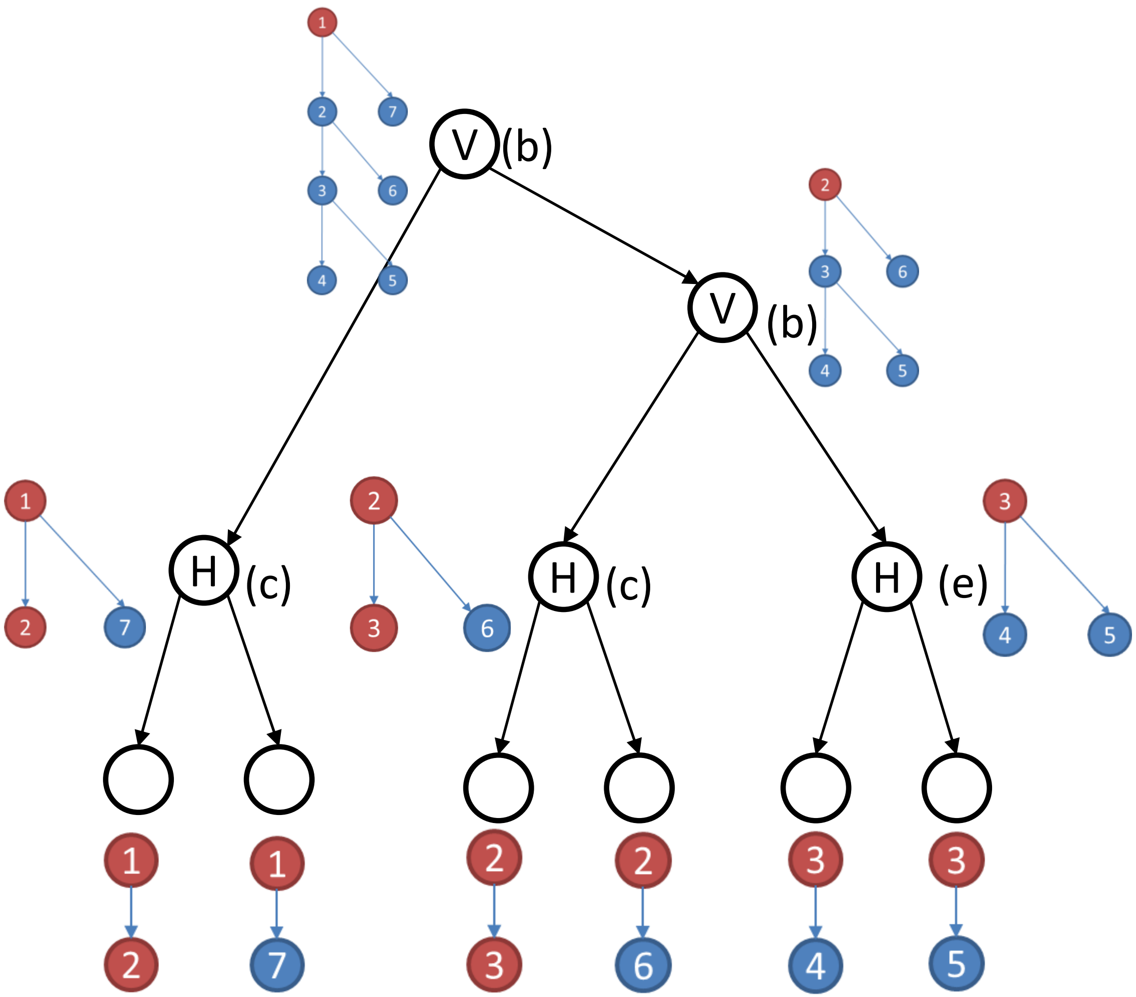


Top tree

- Example:



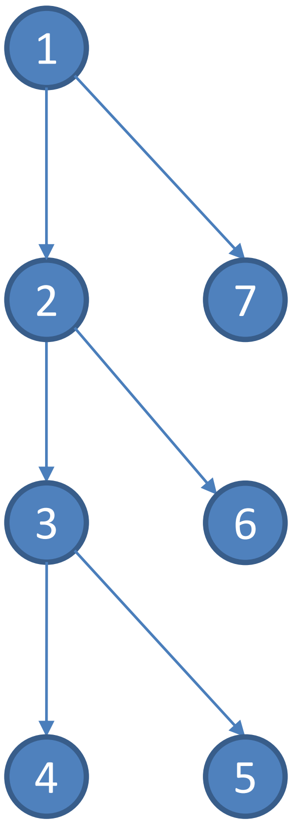
Input tree T



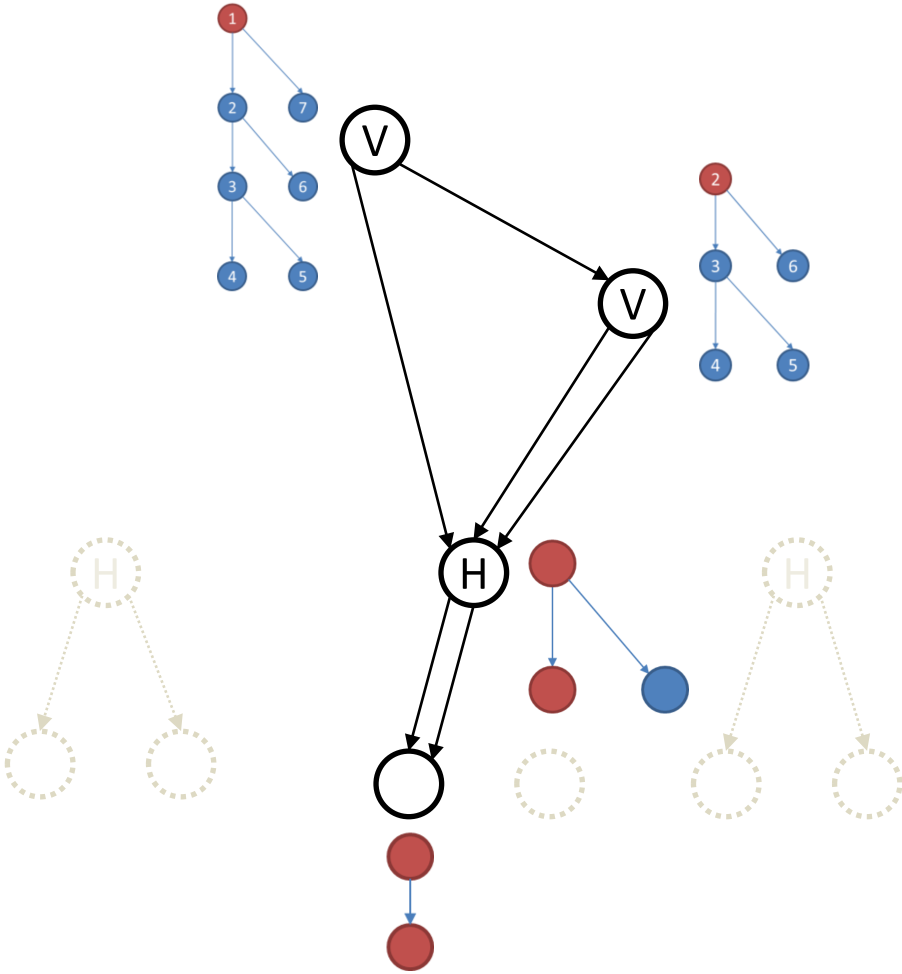
top tree $\mathcal{T}$

Top tree

- Example:



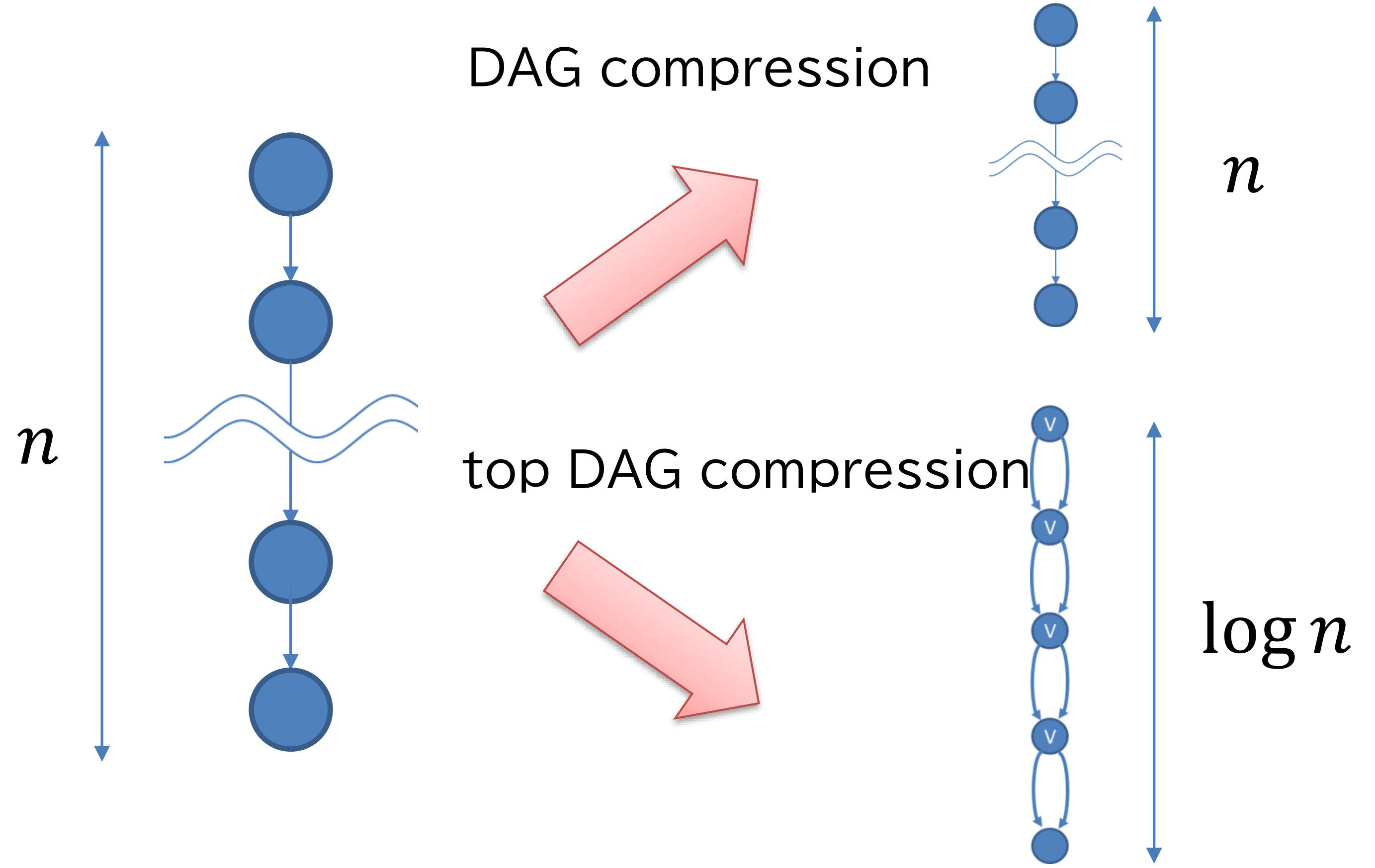
Input tree T



top DAG $\mathcal{TD}$

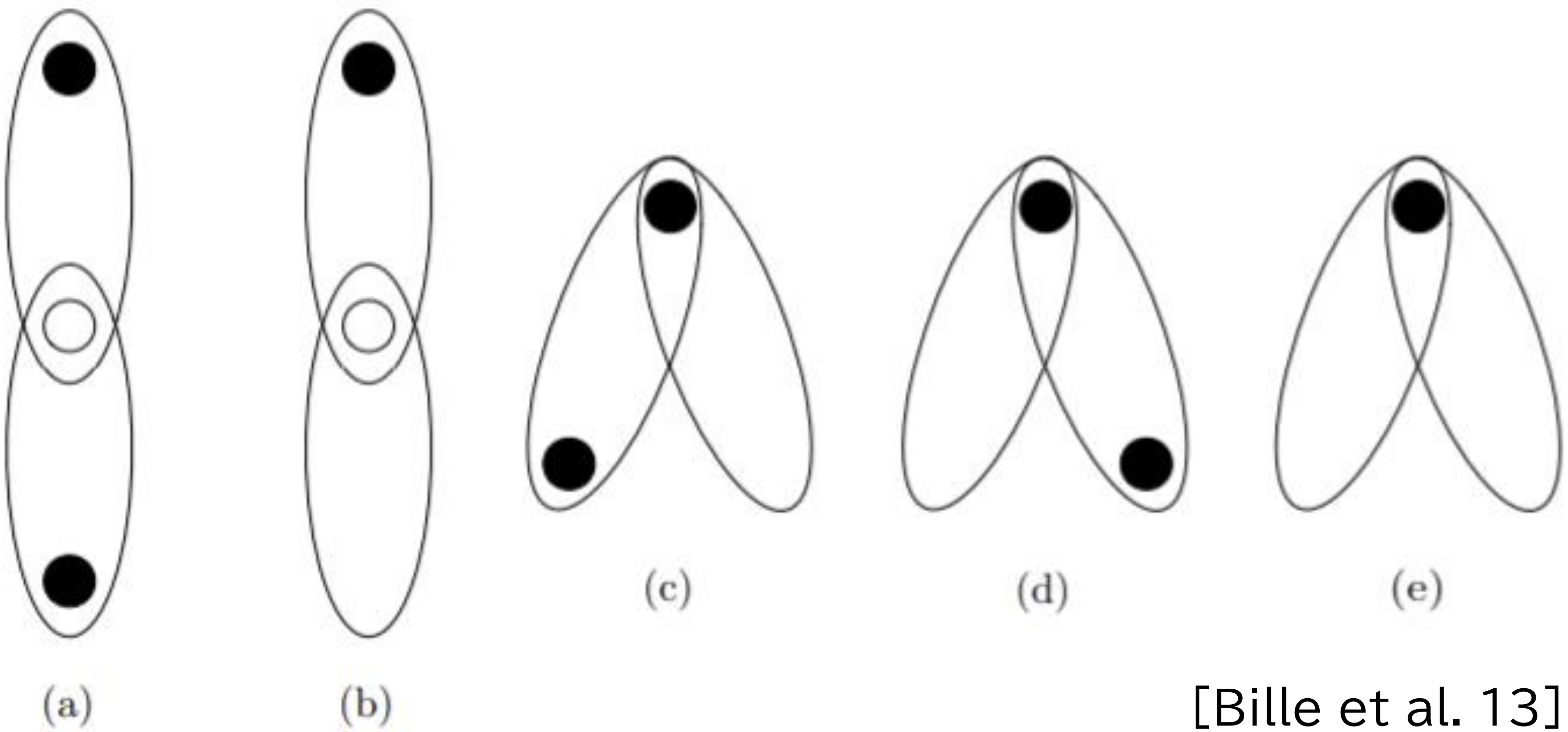
Advantage of top DAG

- Top DAG compression
  allows sharing the same substructure
  that appear at different height



Two types of merging

- Vertical merge: (a), (b)
- Horizontal merge: (c), (d), (e)

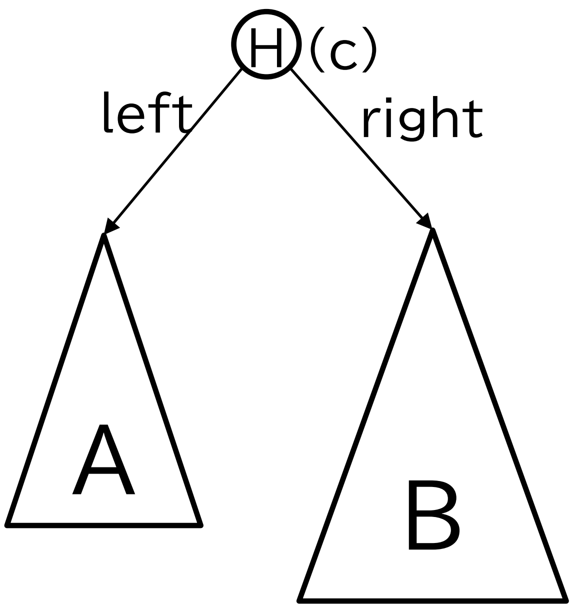


[Bille et al. 13]

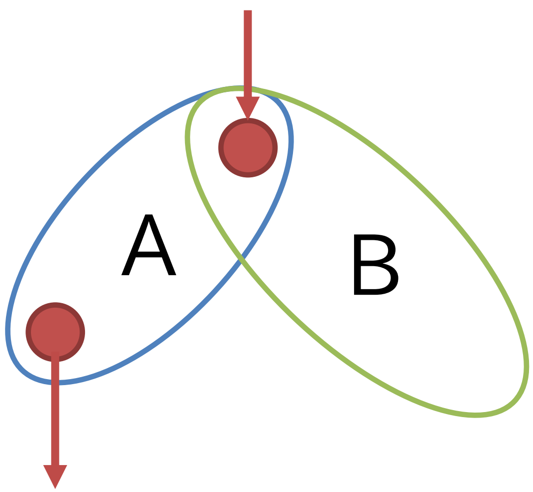
Horizontal merge

- Merge two clusters
  that have the same node
  as their top boundary nodes

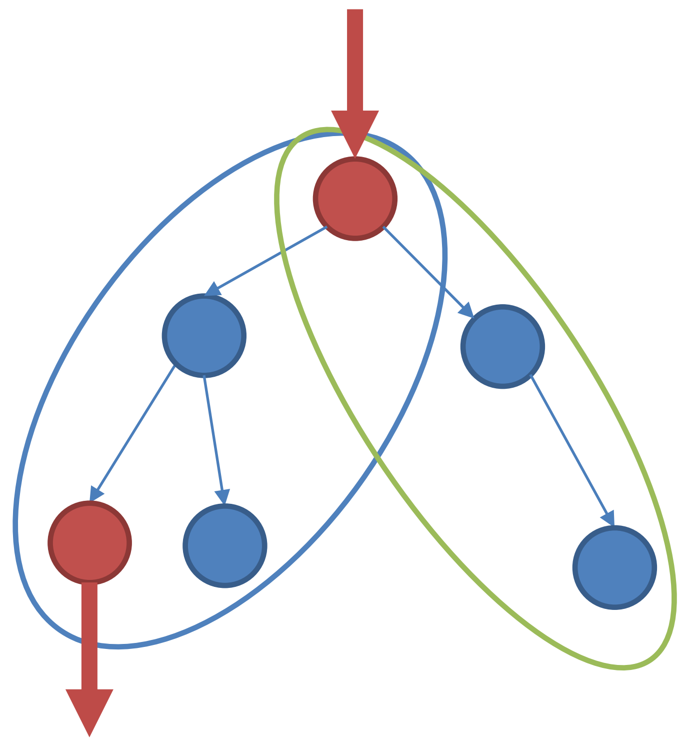
top tree



Corresponding clusters

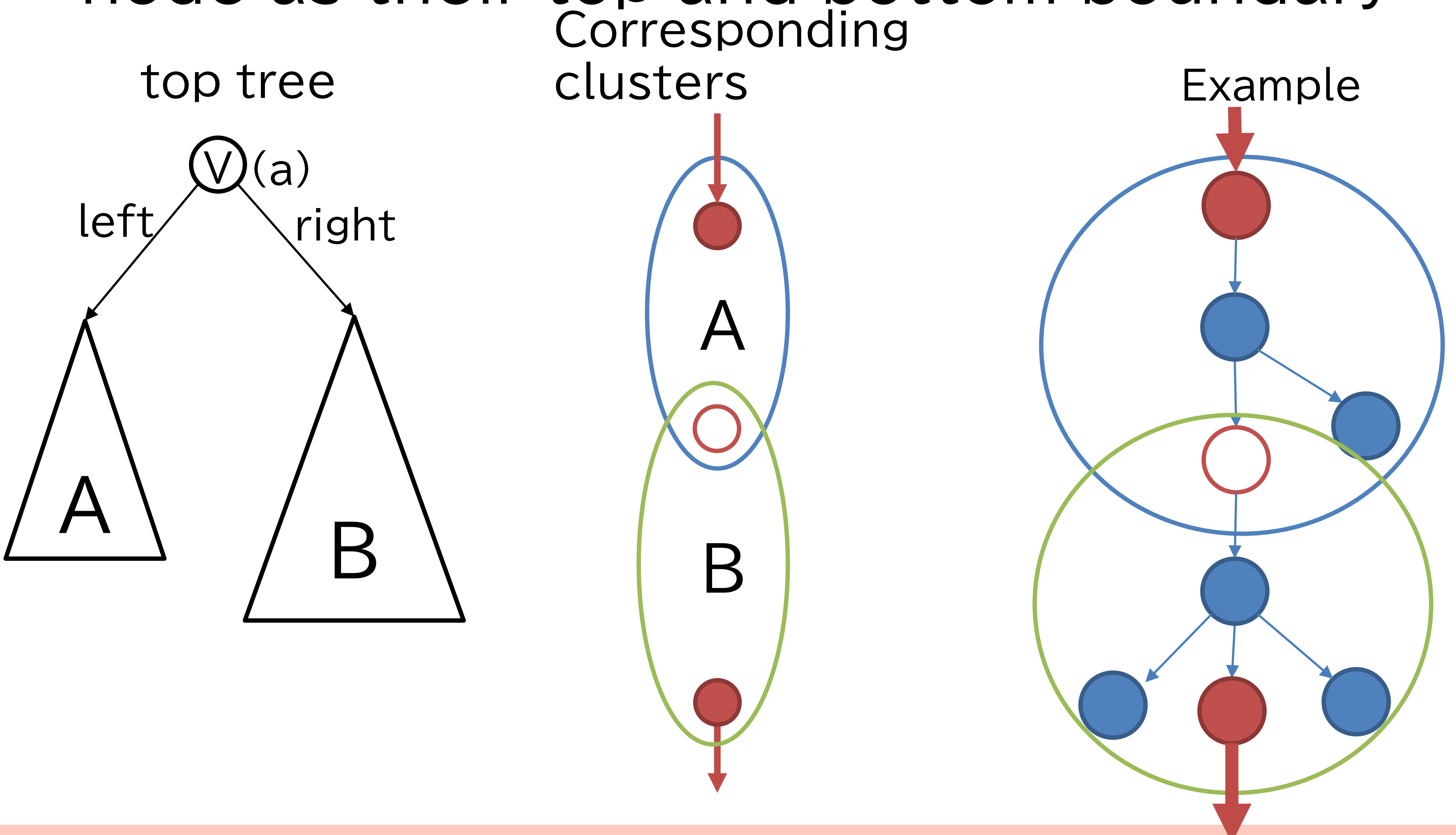


Example



Vertical merge

- Merge two clusters that have the same node as their top and bottom boundary

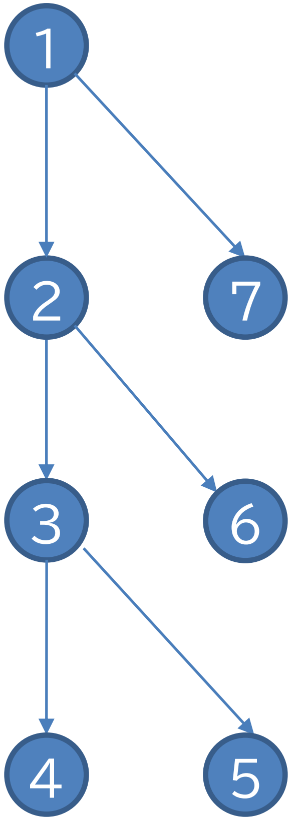


Top tree construction

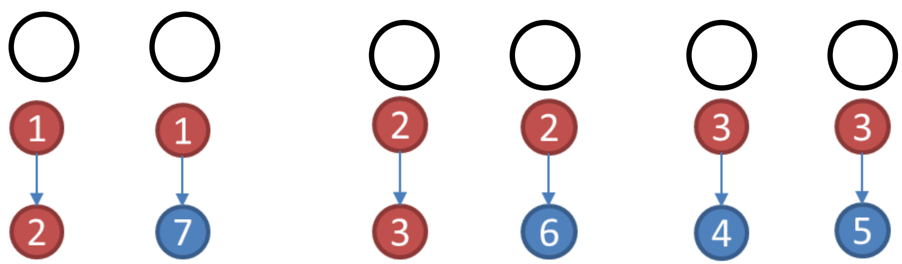
- Top tree is not uniquely determined from the input tree
- Greedy construction
  - Repeat 1—3 until the tree T become 1 edge
  - 1. Choose pairs of clusters that
    can be horizontally merged as much as possible
  - 2. Choose pairs of clusters that
    can be vertically merged from remaining nodes as much as possible
  - 3. Merge the all pairs chosen at 1 and 2

Greedy construction

- Example



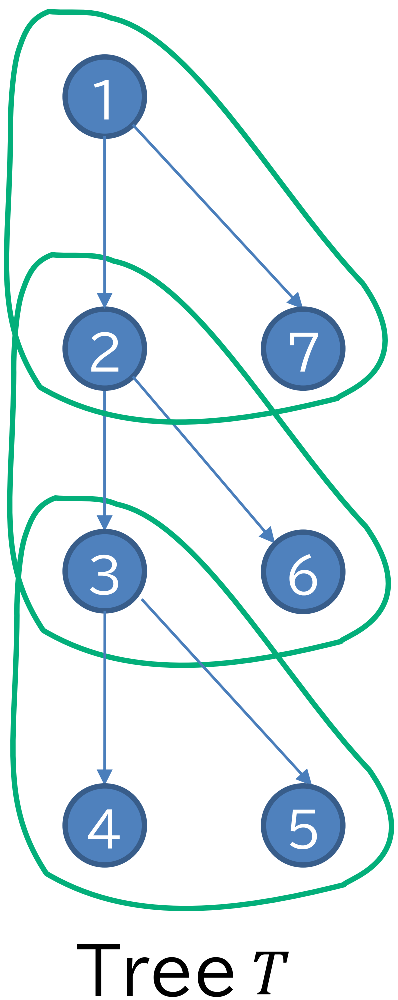
Tree T

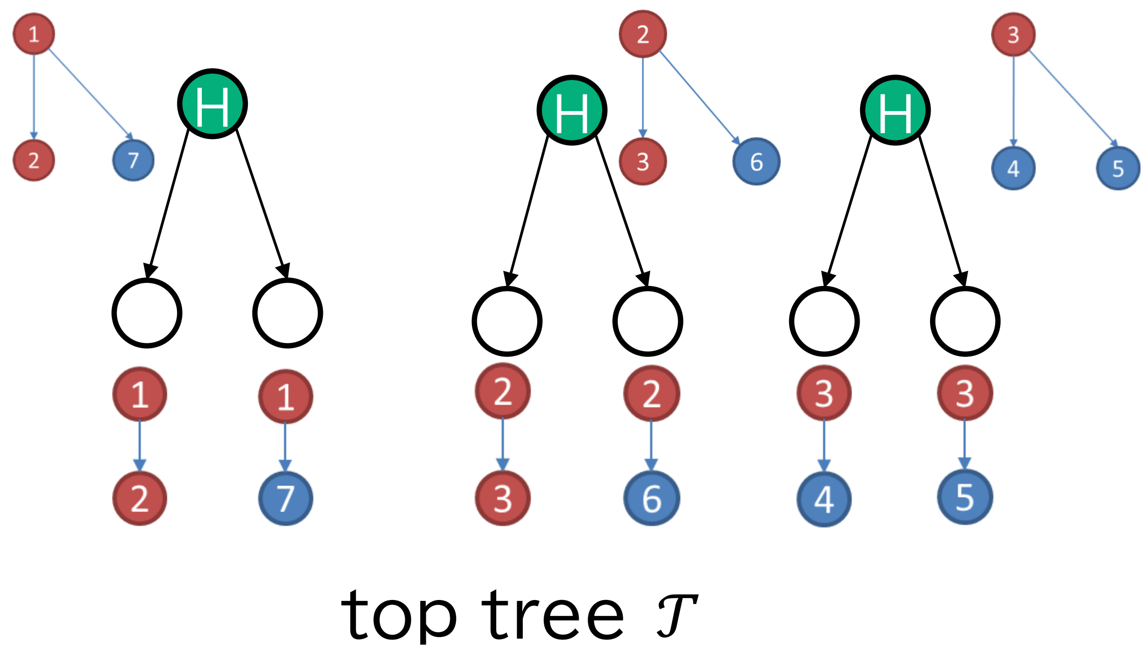


top tree $\mathcal{T}$

Greedy construction

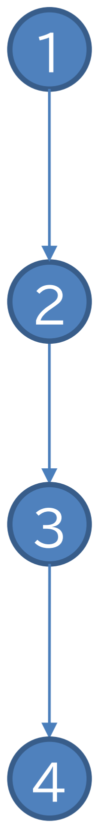
- Example



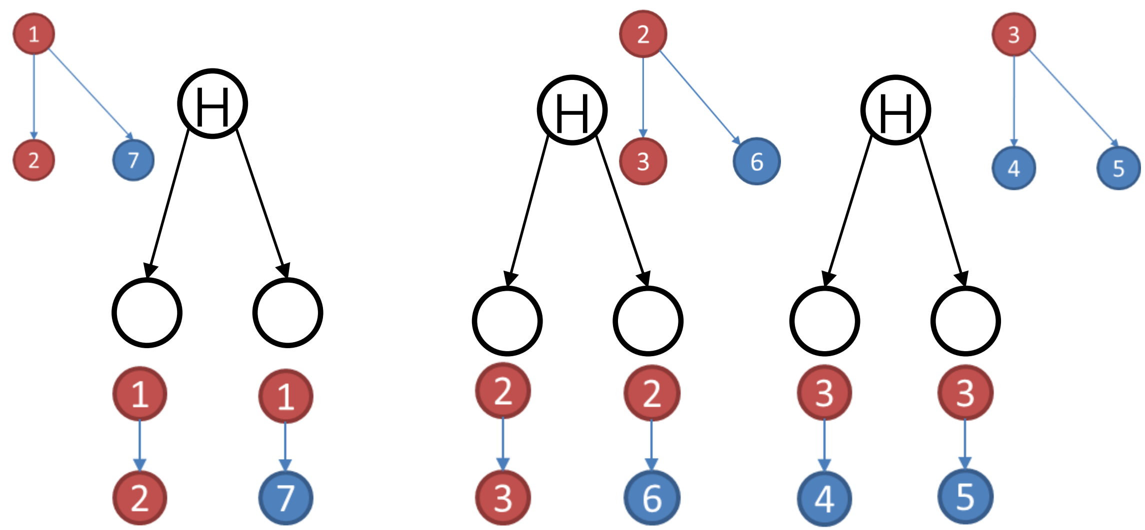


Greedy construction

- Example



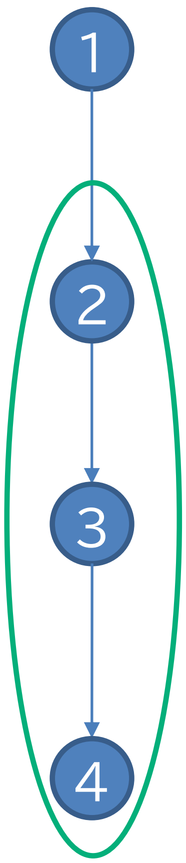
Tree T



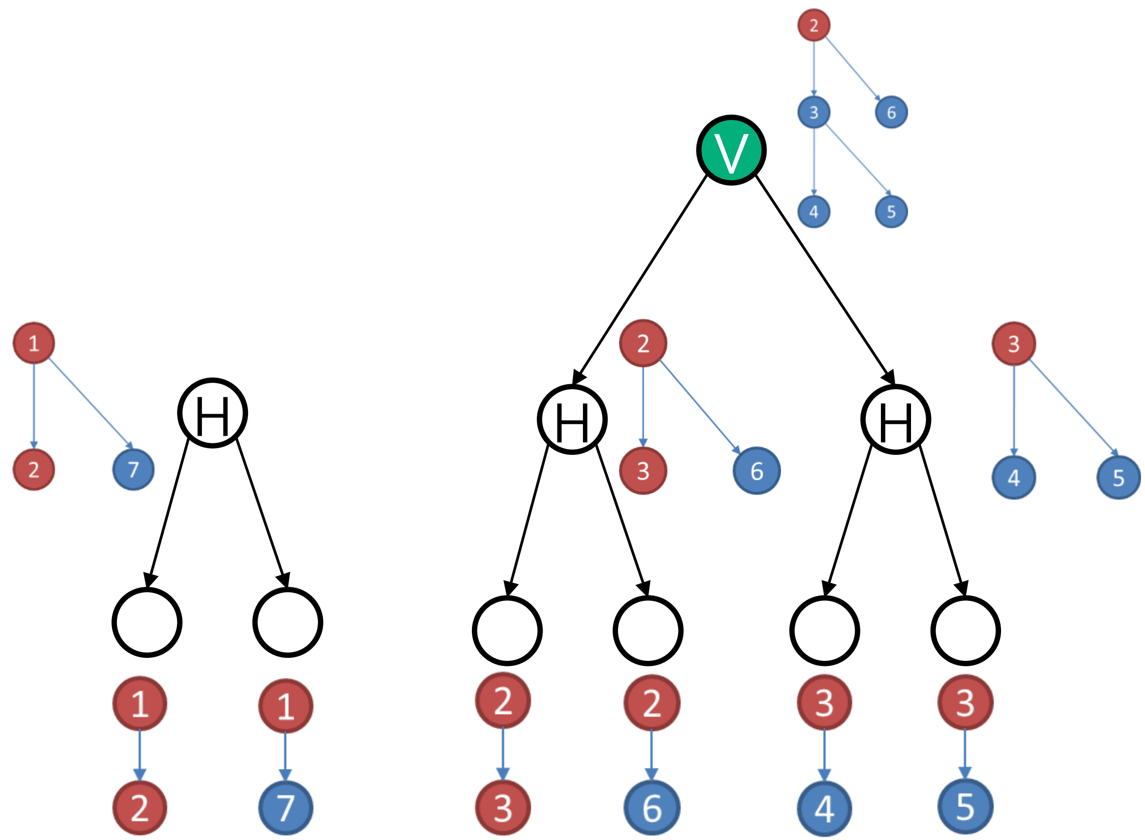
top tree $\mathcal{T}$

Greedy construction

- Example



Tree T

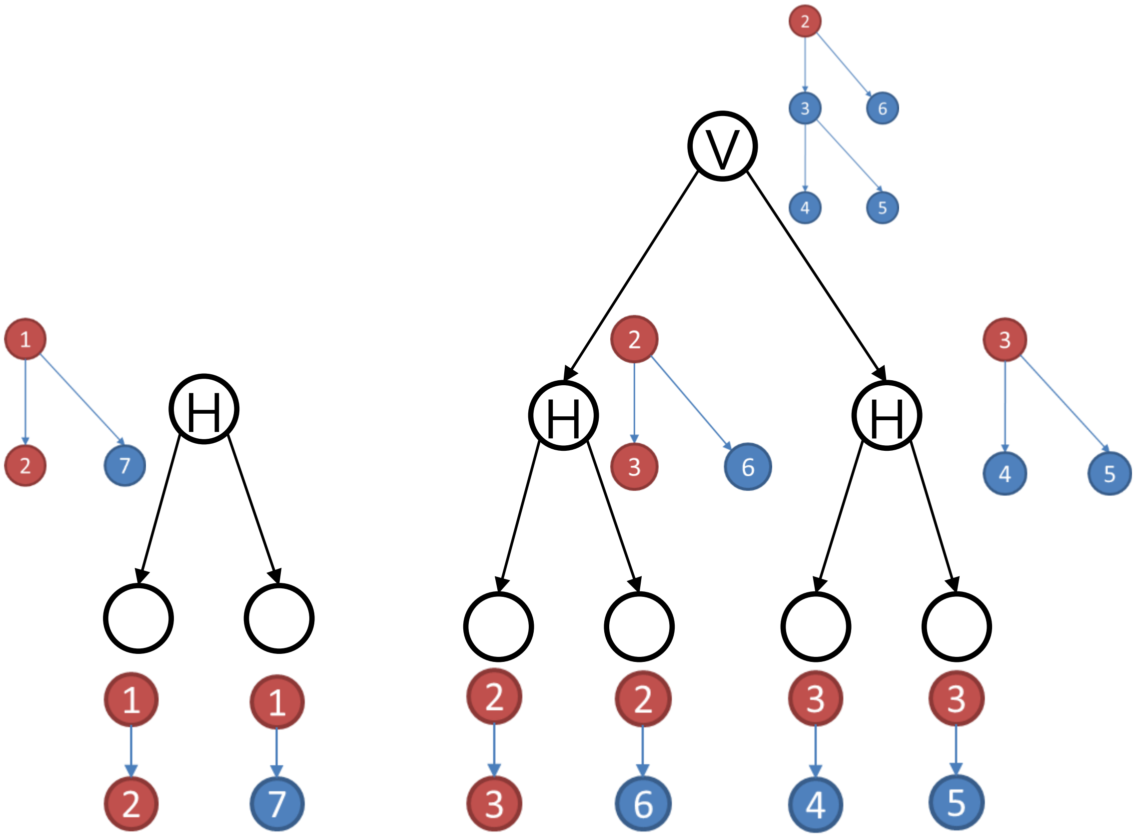


top tree $\mathcal{T}$

Greedy construction

- Example

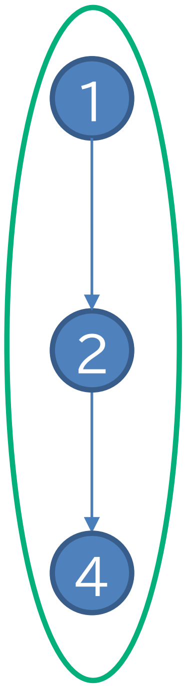
Tree T



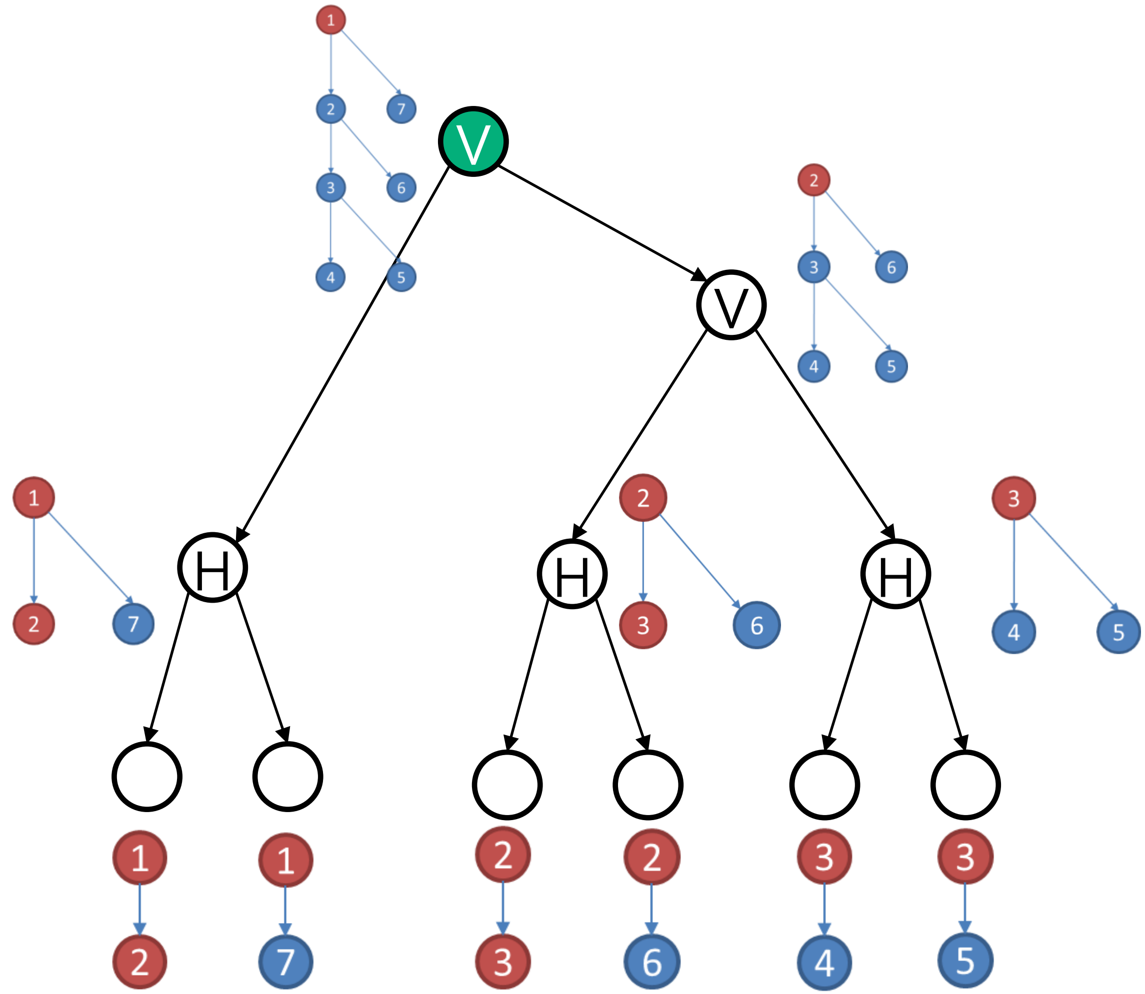
top tree $\mathcal{T}$

Greedy construction

- Example



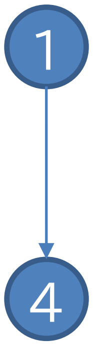
Tree T



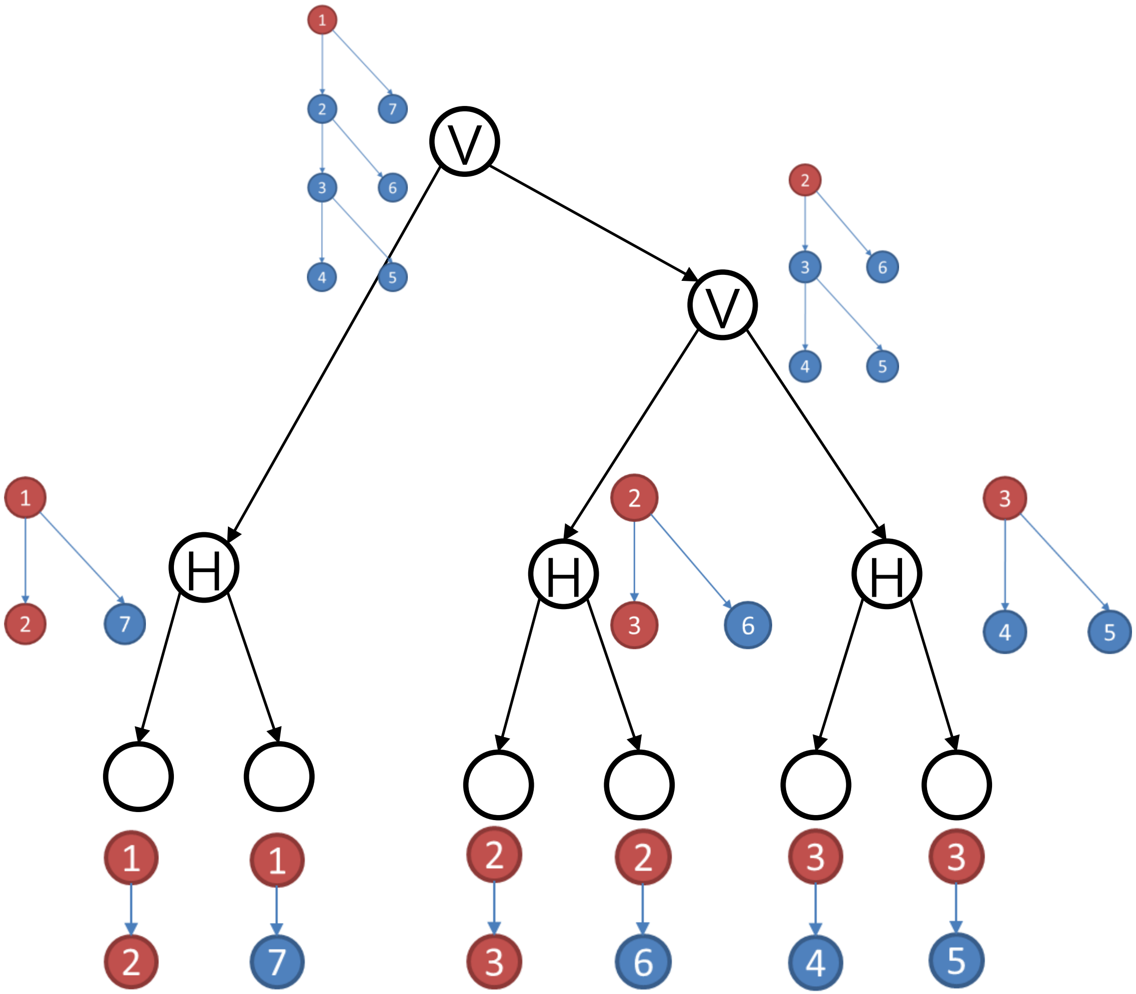
top tree $\mathcal{T}$

Greedy construction

- Example



Tree T



top tree $\mathcal{T}$

Complexity: Greedy method

- n: #node of an input tree, σ: #label

Theorem
[Bille et al. 13]
The height of the top tree made by greedy construction is $O(\log n)$

Theorem
[Hübchle-Schneider, Raman 15]
The number of nodes of top DAG obtained after DAG compression to the top tree made by greedy construction is $O((n \log \log_\sigma n) / \log_\sigma n)$

Operations on top DAG

- Following operations are in $O(\log n)$ time
  - (x: x-th node in DFS, $T(x)$: a subtree rooted by x)
  - $\mathrm{access}(x)$: label of x
  - $\mathrm{parent}(x)$: preorder of the parent of x
  - $\mathrm{depth}(x)$: depth of x
  - $\mathrm{height}(x)$: height of x
  - $\mathrm{size}(x)$: number of nodes in $T(x)$
  - $\mathrm{firstchild}(x)$: preorder of the first child of x
  - $\mathrm{nextsibling}(x)$: preorder of the next sibling of x
  - $\mathrm{la}(x, i)$: preorder of i-th ancestor of x
  - $\mathrm{nca}(x, y)$: preorder of nearest common ancestor of x and y

Proposed method

Top ZDD

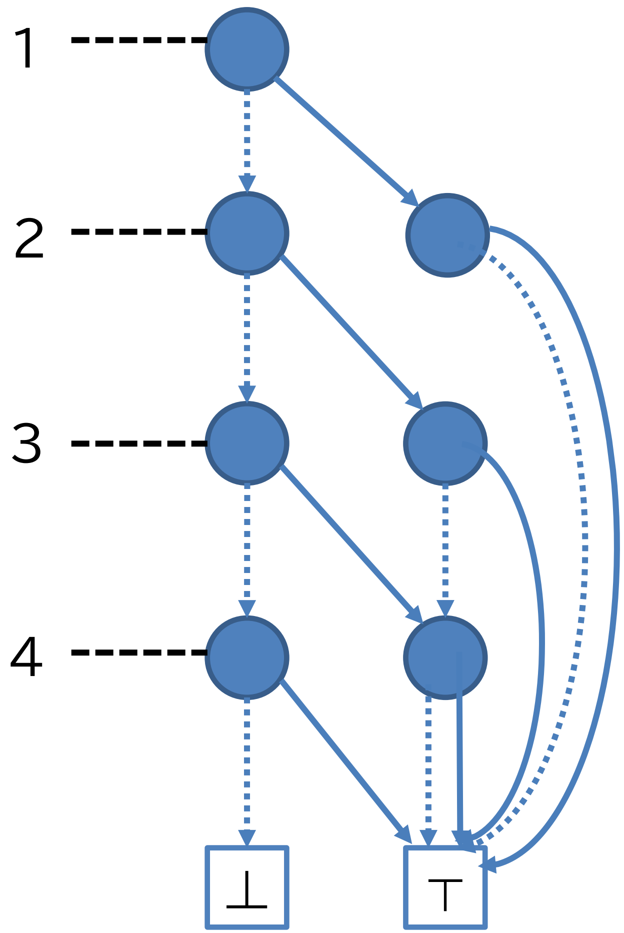
Construction algorithm
Experiment

Construction of top ZDD

- 1. Find a spanning tree from input ZDD
  - The edges not included in the spanning tree is called non tree edges
- 2. Transform the spanning tree to a top tree by greedy construction
- 3. For each non tree edge, store the edge at the nearest common ancestor of the source node and the destination node (Edges point sink nodes are exception)
- 4. Share equivalent subtrees

Example of construction

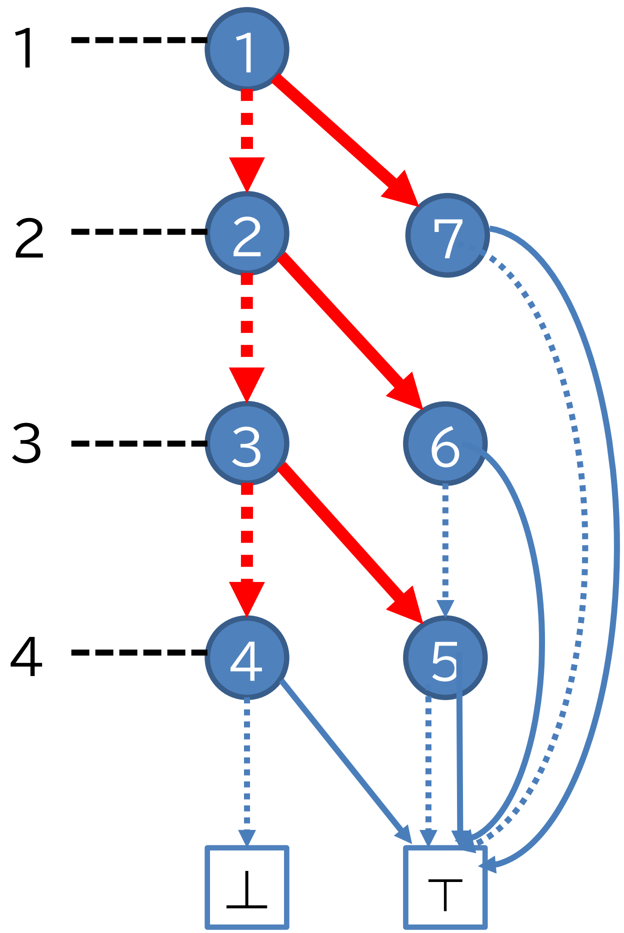
- Step 0. Input



Original ZDD

Example of construction

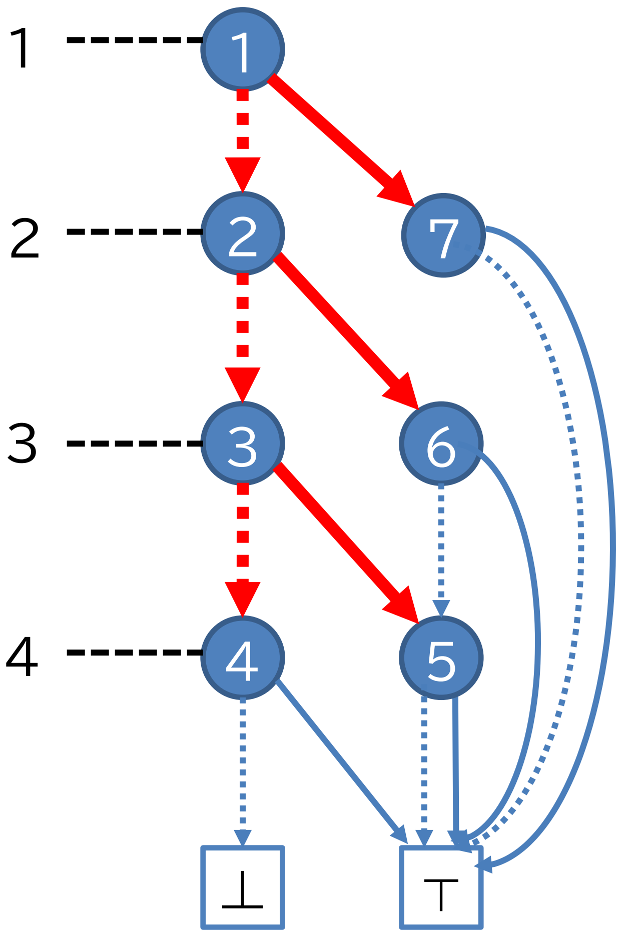
- Step 1. Find a spanning tree



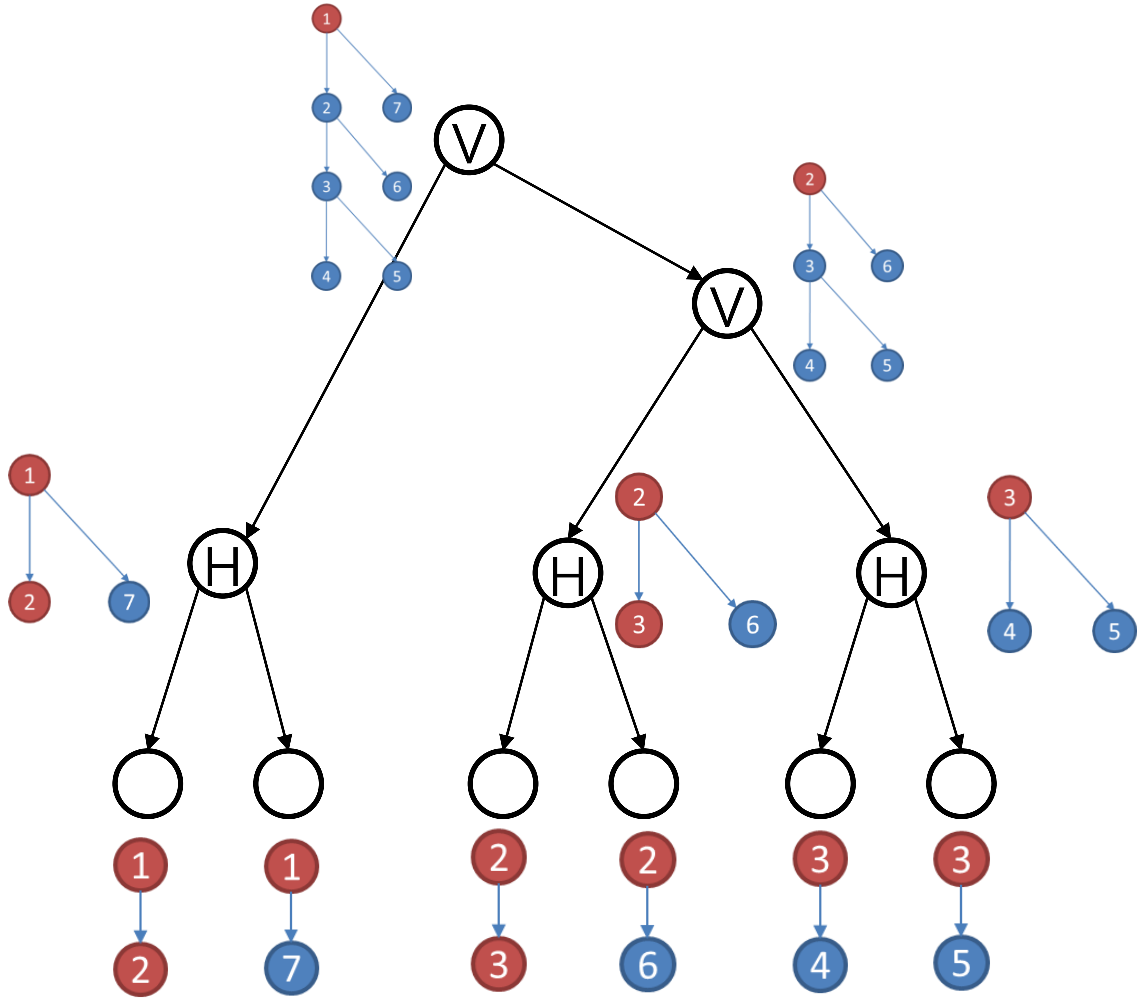
Original ZDD

Example of construction

- Step 2. Construct a top tree

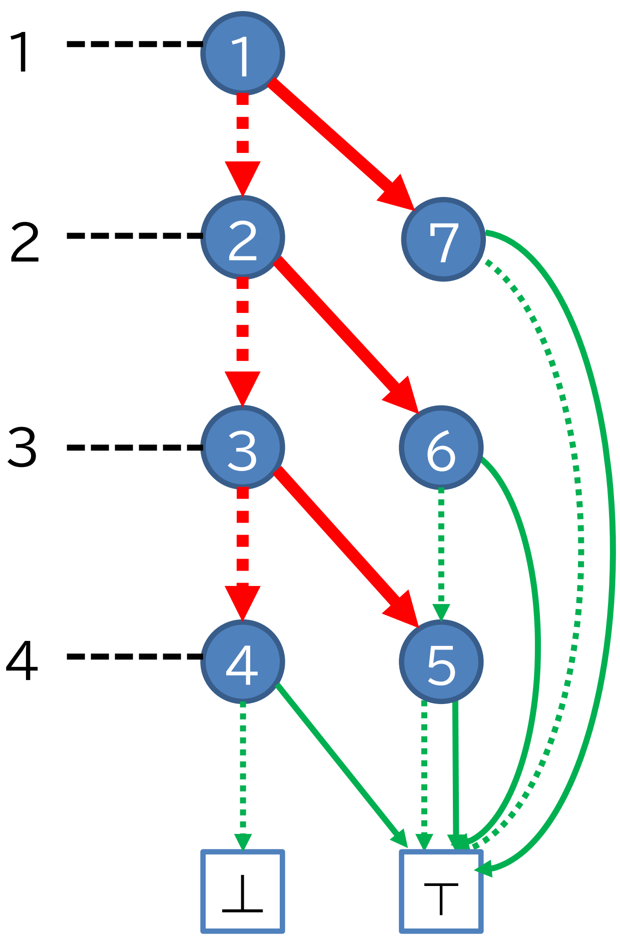


Original ZDD

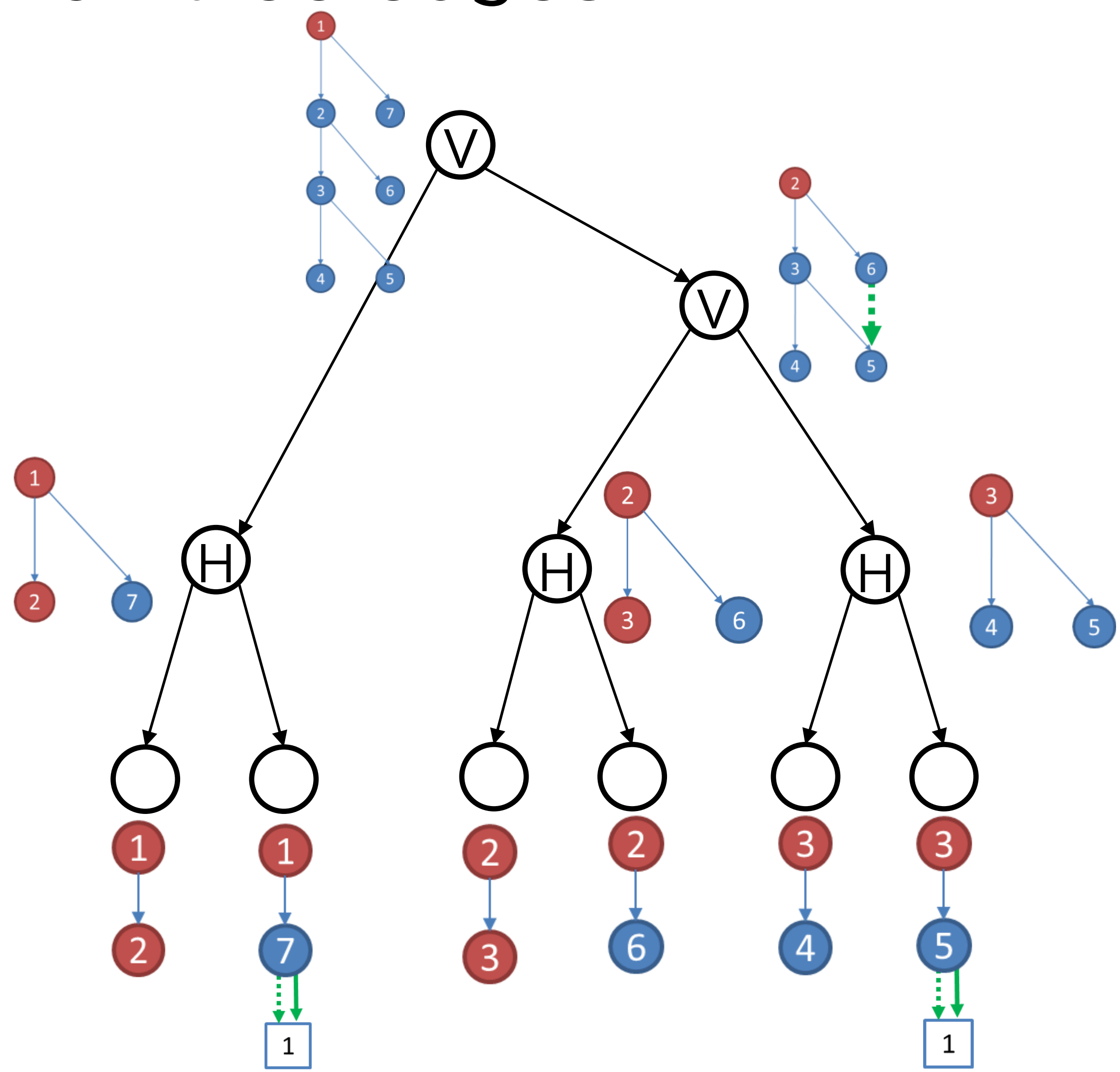


Example of construction

- Step 3. Store non tree edges

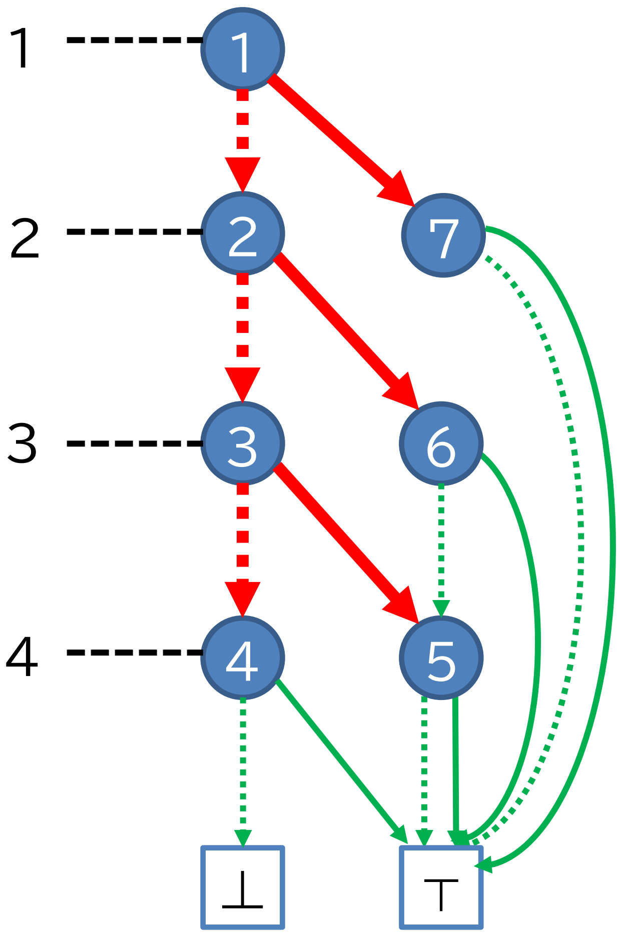


Original ZDD

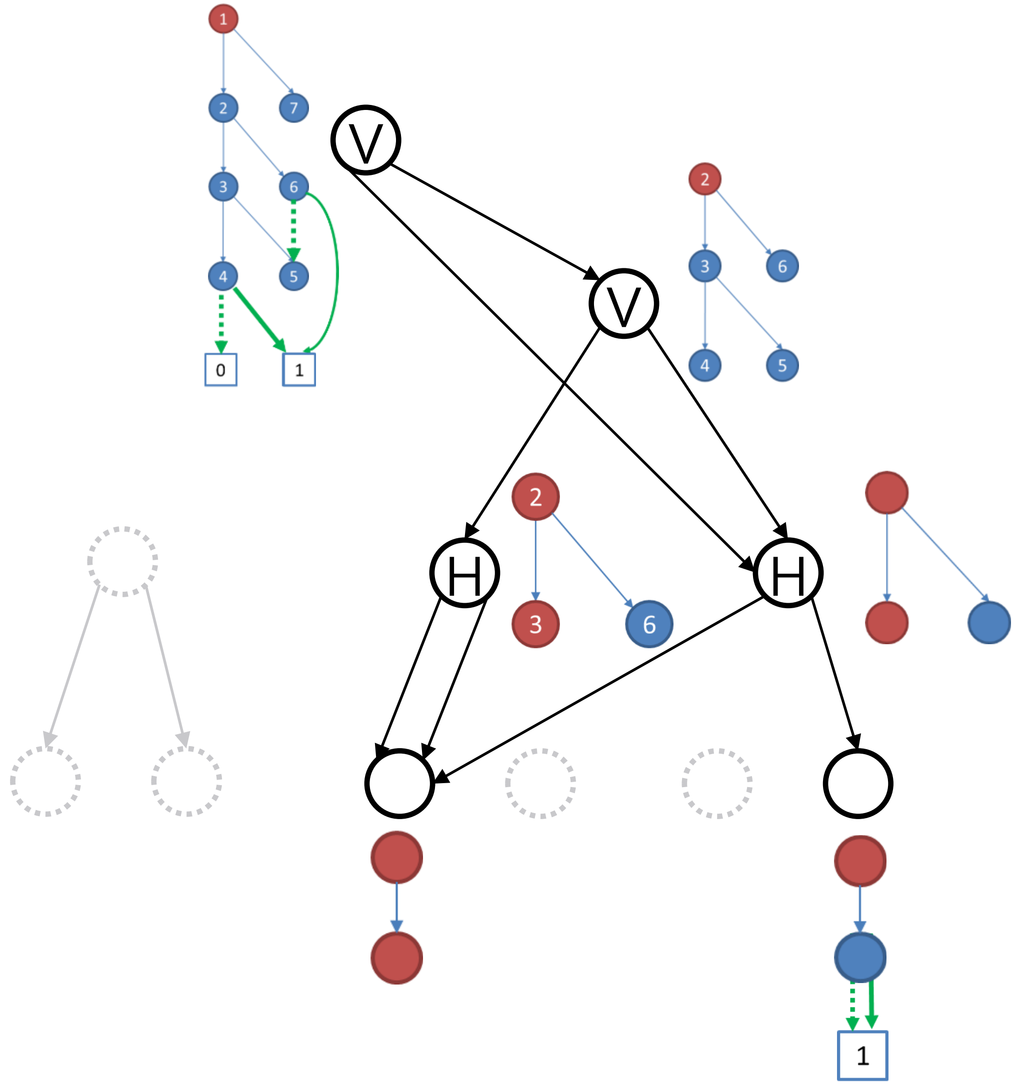


Example of construction

- Step 4. Share equivalent subtrees



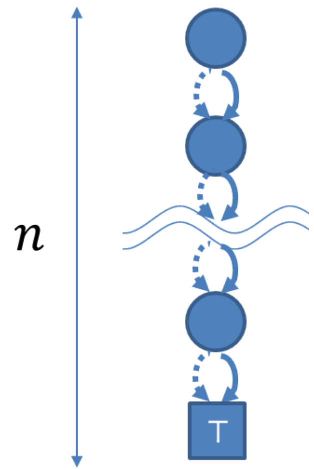
Original ZDD

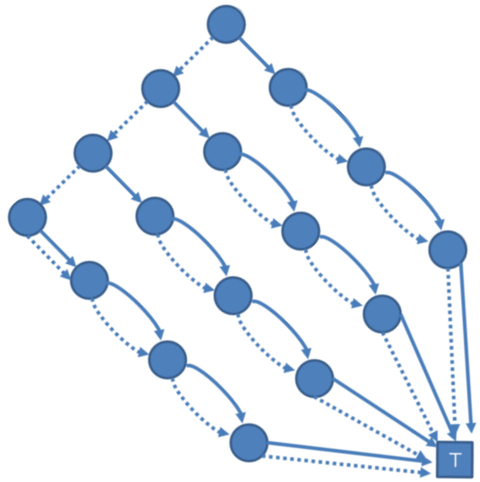


Theoretical results

Theorem
Memory usage of top ZDD
is O(log n) in the best case

- Examples





Theorem
Edge traversal is $O(\log^2 n)$ time

Experiment

- Compared data structures
  - Top ZDD: memory usage (byte)
  - DenseZDD: memory usage (byte)
    - Static ZDD using succinct data structure[Denzumi et al. 14]
  - ZDD: $(2n \log n + n \log \sigma)$ (n: #node, σ: #label) (byte)
- Data sets
  - $\{S \subseteq U \mid |S| \leq B\}$, where $|U| = A$
  - $\{S \subseteq U \mid {}^{\forall}e \in S, {}^{\exists}f \in S \text{ s.t. } |e - f| \leq B\} \cup U$, where $U = \{1, \ldots, A\}$
  - 2^U, where $|U| = A$
  - Solutions of knapsack problems
  - Sets of matching edges of graphs
  - Frequent item sets

Experimental results 1/6

- Data: For an underlying set U of size A,

$$\{S \subseteq U \mid |S| \leq B\}$$

	top ZDD	DenseZDD	$(2n \log n + n \log c)/8$
$A = 100, B = 50$	3,823	9,544	9,882
$A = 400, B = 200$	13,614	146,550	206,025
$A = 1000, B = 500$	43,151	966,519	1,440,375

(bytes)

- Top ZDDs are 2—20 times smaller than DenseZDDs

Experimental results 2/6

- Data: For an underlying set U of size A,

$$\{S \subseteq U \mid {}^\forall e \in S, {}^\exists f \in S \text{ s.t. } |e - f| \leq B\} \cup U$$

	top ZDD	DenseZDD	$(2n \log n + n \log c)/8$
$A = 500, B = 250$	2,431	227,798	321,594
$A = 1000, B = 500$	2,511	321,594	1,440,375

(bytes)

- Top ZDDs are 100—125 times smaller than DenseZDDs

Experimental results 3/6

- Data: For an underlying set U of size A, 2^U

	top ZDD	DenseZDD	$(2n \log n + n \log c)/8$
A =1000	2,254	4,185	3,750
$A = 50000$	2,464	178,764	300,000

(bytes)

- Top ZDDs are highly effective because the inputs have simple structure

Experimental results 4/6

- Data: Solutions of knapsack problems
  - $w_i \in [1, W]$: random weight ($w_i \geqq w_{i+1}$), C: capacity

	top ZDD	DenseZDD	$(2n \log n + n \log c)/8$
$A = 100, W = 1000,$ $C = 10000$	1,658,494	1,730,401	2,444,405
$A = 200, W = 100,$ $C = 5000$	1,032,596	1,516,840	2,181,688
$A = 1000, W = 100,$ $C = 1000$	2,080,925	2,929,191	4,491,025
$A = 5000, W = 100,$ $C = 200$	1,135,613	1,740,841	2,884,279
$A = 1000, W = 10,$ $C = 1000$	1,382,933	2,618,970	3,990,350
$A = 1000, W = 100,$ $C = 1000$	565,500	656,728	1,056,907

- Top ZDDs are better than DenseZDDs

Experimental results 5/6

- Data: Sets of matching edges of graphs

	top ZDD	DenseZDD	$(2n\log n + n\log c)/8$
8×8 grid graph	12,196	16,150	18,014
Perfect graph K_{12}	23,038	16,304	25,340
"Interroute"	30,780	39,831	50,144

(bytes)

(*"Interroute"* : a graph of real network
http://www.topology-zoo.org/dataset.html)

- Top ZDDs lose in one case,
  but not 1.5 times bigger than DenseZDD

Experimental results 6/6

- Data: Frequent item sets (p: threshold)

	top ZDD	DenseZDD	$(2n \log n + n \log c)/8$
"mushroom" ($p = 0.001$)	104,580	91,757	123,576
"retail" ($p = 0.00025$)	59,854	65,219	62,766
T40I10D100K" ($p = 0.005$)	224,378	188,400	248,656

(bytes)

(Data are from http://fimi.uantwerpen.be/data/)

- Not so big differences between top ZDDs and DenseZDDs

Conclusion

- Proposed compression method for ZDD
  - Expand top tree compression
  - Choose a spanning tree from an input ZDD
  - Unlike DenseZDD, top ZDD does not separate the spanning tree and non-tree edges
  - Experiments showed efficiency of top ZDDs
- Future work
  - Dynamic programming on top ZDDs
  - Faster operations on top ZDDs
  - Finding better spanning trees for compression
  - Complexity of finding optimal spanning tree
  - Applying proposed method for general DAGs

Thank you
for listening!