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# Pattern Discovery in Colored Strings

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# Motivations – Assertion mining

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Embedded Systems are everywhere

The design of embedded systems requires to evaluate the correctness of its functionalities. Usually done using **assertions** (Logic formulae).

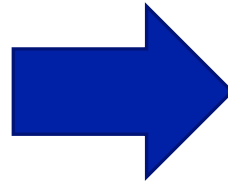
Typically written by hand by the designers. It might take months to find a small and effective set of assertions.

Automatic extraction of **assertions** from simulation traces.

# Motivations – Assertion mining

Simulation trace

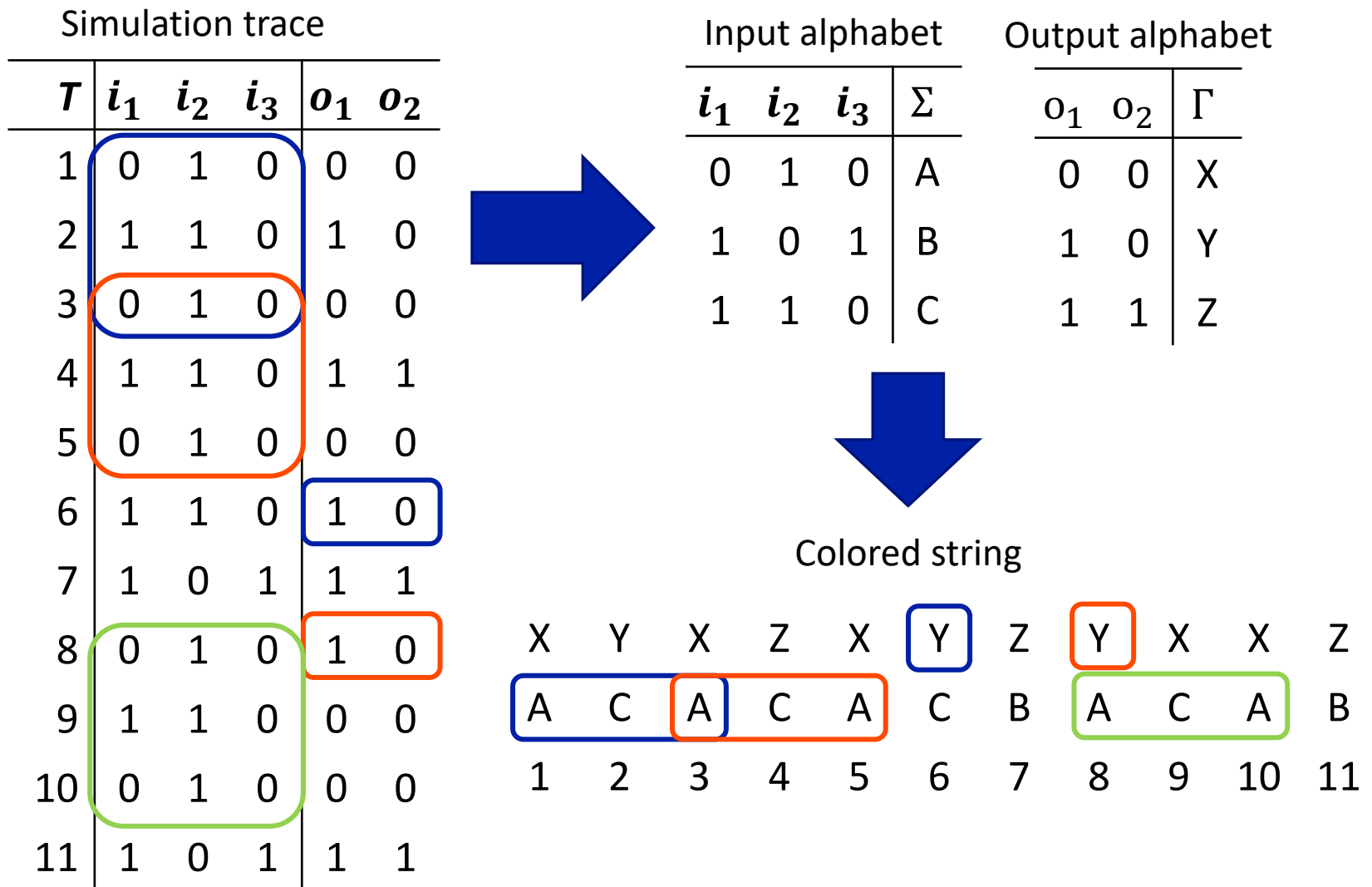
| $T$ | $i_1$ | $i_2$ | $i_3$ | $o_1$ | $o_2$ |
|-----|-------|-------|-------|-------|-------|
| 1   | 0     | 1     | 0     | 0     | 0     |
| 2   | 1     | 1     | 0     | 1     | 0     |
| 3   | 0     | 1     | 0     | 0     | 0     |
| 4   | 1     | 1     | 0     | 1     | 1     |
| 5   | 0     | 1     | 0     | 0     | 0     |
| 6   | 1     | 1     | 0     | 1     | 0     |
| 7   | 1     | 0     | 1     | 1     | 1     |
| 8   | 0     | 1     | 0     | 1     | 0     |
| 9   | 1     | 1     | 0     | 0     | 0     |
| 10  | 0     | 1     | 0     | 0     | 0     |
| 11  | 1     | 0     | 1     | 1     | 1     |



Simulation trace

| $T$ | $i_1$ | $i_2$ | $i_3$ | $o_1$ | $o_2$ |
|-----|-------|-------|-------|-------|-------|
| 1   | 0     | 1     | 0     | 0     | 0     |
| 2   | 1     | 1     | 0     | 1     | 0     |
| 3   | 0     | 1     | 0     | 0     | 0     |
| 4   | 1     | 1     | 0     | 1     | 1     |
| 5   | 0     | 1     | 0     | 0     | 0     |
| 6   | 1     | 1     | 0     | 1     | 0     |
| 7   | 1     | 0     | 1     | 1     | 1     |
| 8   | 0     | 1     | 0     | 1     | 0     |
| 9   | 1     | 1     | 0     | 0     | 0     |
| 10  | 0     | 1     | 0     | 0     | 0     |
| 11  | 1     | 0     | 1     | 1     | 1     |

# Motivations – Assertion mining



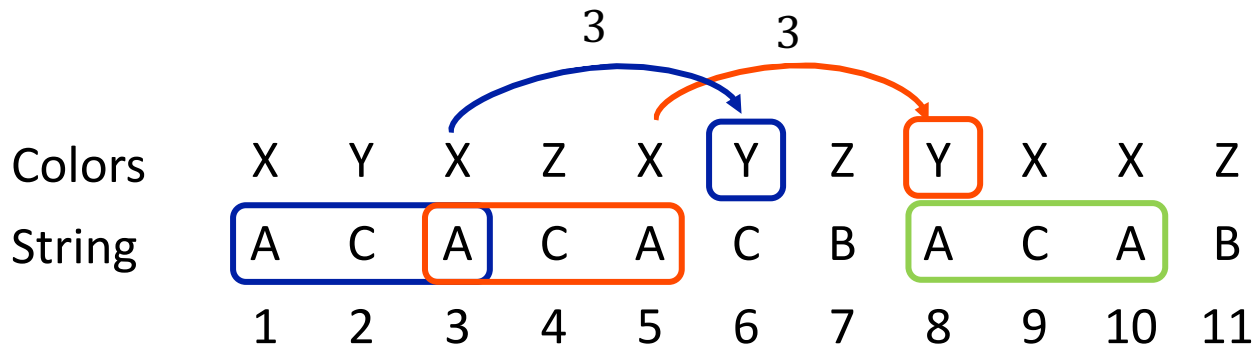
# Colored Strings

## Definition

Colored strings are strings where each character is assigned one of a finite set of colors.

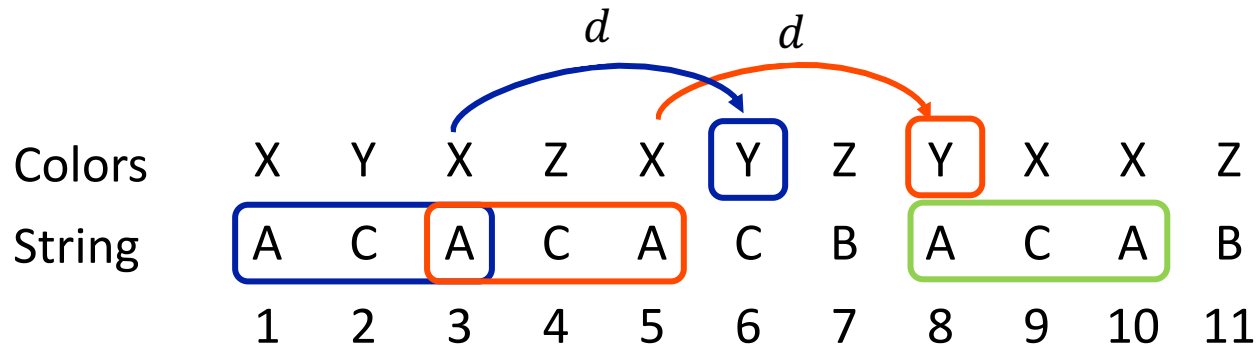
## Objective

We want to find patterns in the string that always occur with the same color at a certain distance.



We say that ACA is (Y,3)-unique.

# Pattern Discovery



We say that ACA is (Y,3)-unique.

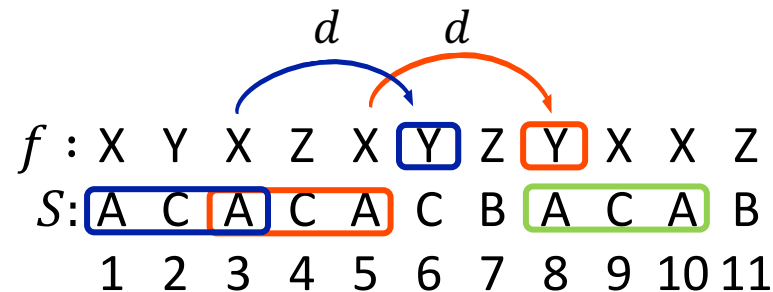
## Problem

Given a colored string  $S$  and a color  $Y$ , report all pairs  $(T,d)$  such that  $T$  is  $(Y,d)$ -unique substring of  $S$ .

## Note

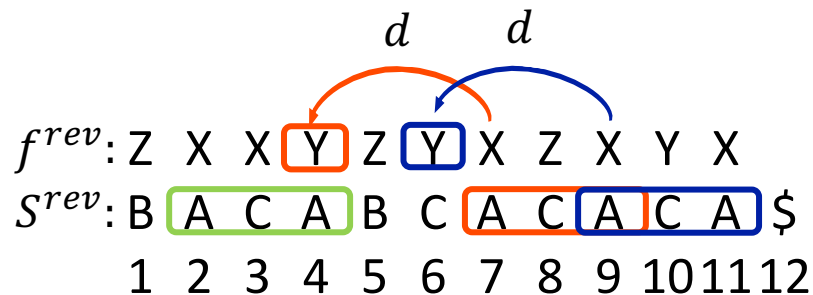
**Although this problem is simpler than the assertion mining problem, the solution to our problem contains all the information, possibly filtered, to recover the desired set of minimal assertions in a second stage.**

# Discovery all $(y, d)$ -unique substrings



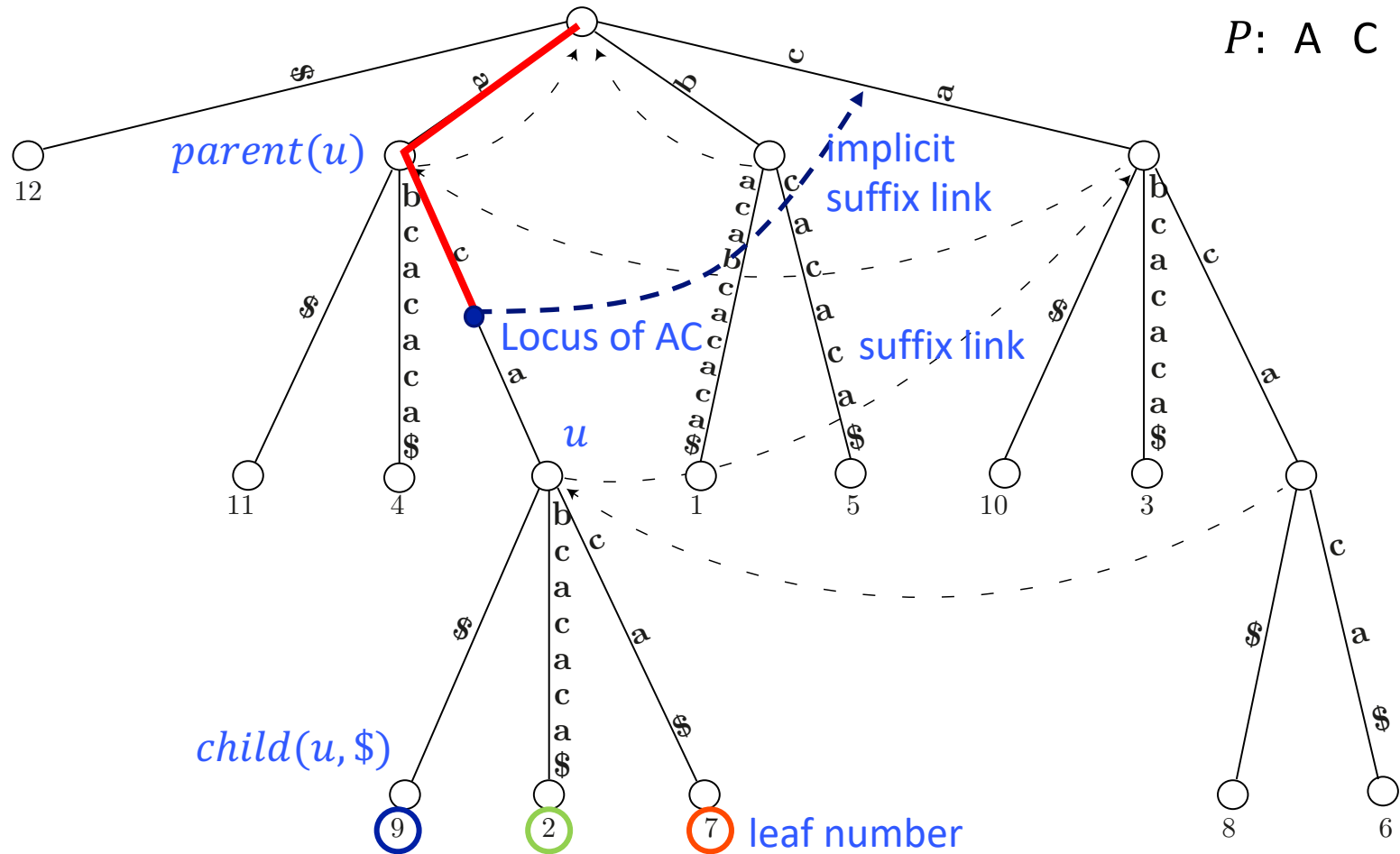
We need to check all occurrences of a substring of  $S$ . To keep the space contained, we use dedicated string data structures, i.e. **Suffix trees**.

Since the delay is measured from the end of the substring, it is convenient to think in terms of prefixes, i.e. **Suffixes of the reverse string**.



# Suffix tree

S: B A C A B C A C A C A \$  
1 2 3 4 5 6 7 8 9 10 11 12

$$P: A \quad C$$


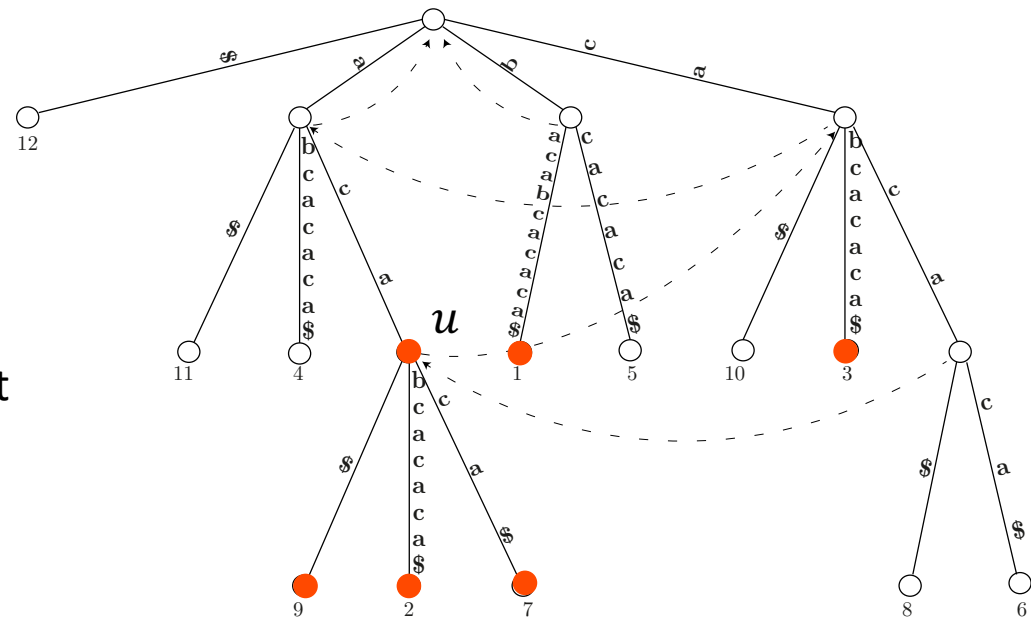


# Discovery all $(y, d)$ -unique substrings

$f : X \ Y \ X \ Z \ X \ \boxed{Y} \ Z \ \boxed{Y} \ X \ X \ Z$   
 $S : \boxed{A \ C} \ \boxed{A \ C \ A} \ C \ B \ \boxed{A \ C \ A} \ B$   
 1 2 3 4 5 6 7 8 9 10 11

$f^{rev} : Z \ X \ X \ \boxed{Y} \ Z \ \boxed{Y} \ X \ Z \ X \ Y \ X$   
 $S^{rev} : B \ \boxed{A \ C \ A} \ B \ C \ \boxed{A \ C} \ \boxed{A \ C \ A} \ \$$   
 1 2 3 4 5 6 7 8 9 10 11 12

1. Build the **suffix tree** of  $S^{rev}$
2. Color a leaf if:
  - Either  $ln \leq d$
  - or  $f(ln - d) = y$
3. Color an internal node if:
  - All children are colored.
4. If a node  $u$  is colored, output all strings represented along the incoming edge of  $u$ .

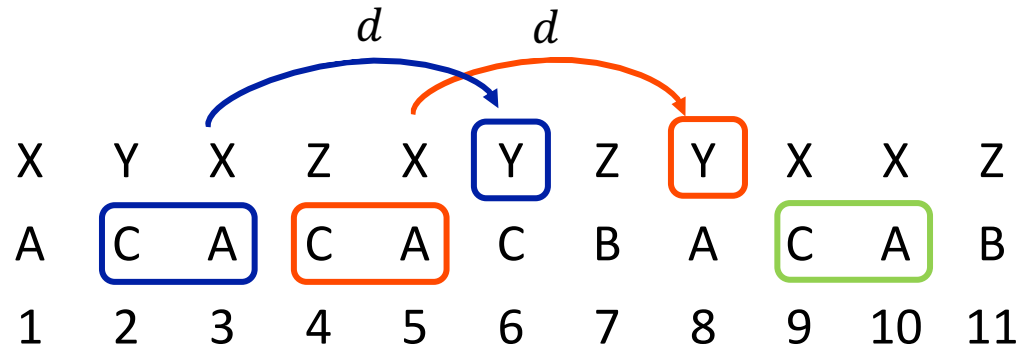


**Output:** ..., CA, ACA, ...

Runs in  $O(n^3)$  time.

$d = 3$   
 $y = Y$

# Minimal $(y, d)$ -unique substrings



We say that CA is **minimal**  $(Y, 3)$ -unique,  
because A is **not**  $(Y, 3)$ -unique and C is **not**  $(Y, 4)$ -unique.

left-minimality

right-minimality

## Problem

Given a colored string  $S$  and a color  $Y$ , report all pairs  $(T, d)$  such that  $T$  is **minimal**  $(Y, d)$ -unique substring of  $S$ .

# Discovery all minimal $(y, d)$ -unique substrings

$f : X \ Y \ X \ Z \ X \ \boxed{Y} \ Z \ \boxed{Y} \ X \ X \ Z$      $f^{rev} : Z \ X \ X \ \boxed{Y} \ Z \ \boxed{Y} \ X \ Z \ X \ Y \ X$   
 $S : A \ \boxed{C} \ \boxed{A} \ \boxed{C} \ \boxed{A} \ C \ B \ A \ \boxed{C} \ \boxed{A} \ B$      $S^{rev} : B \ \boxed{A} \ \boxed{C} \ A \ B \ C \ \boxed{A} \ \boxed{C} \ \boxed{A} \ \boxed{C} \ A \ \$$   
 1 2 3 4 5 6 7 8 9 10 11                      1 2 3 4 5 6 7 8 9 10 11 12

We say that CA is **minimal** (Y,3)-unique, because

1. A is **not** (Y,3)-unique **and**
2. C is **not** (Y,4)-unique.

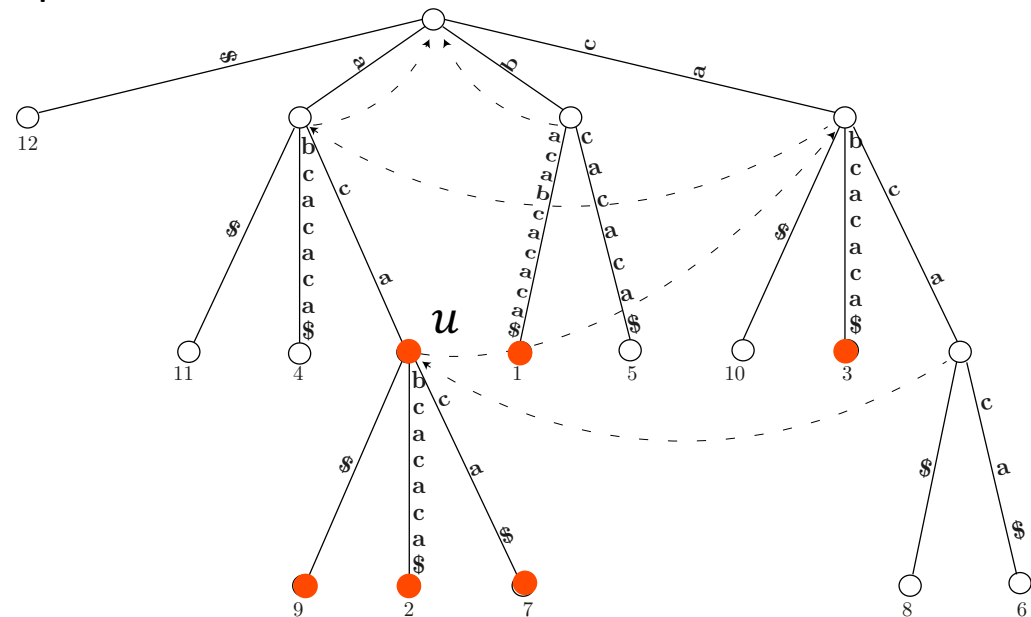
(left minimality)

Parent of AC is not colored.

Suffix link of AC is not colored for  $d = 4$ . (right minimality)

Process  $d$  from 11 downto 0

**Output:** ..., CA, ...



Runs in  $O(n^2)$  time.

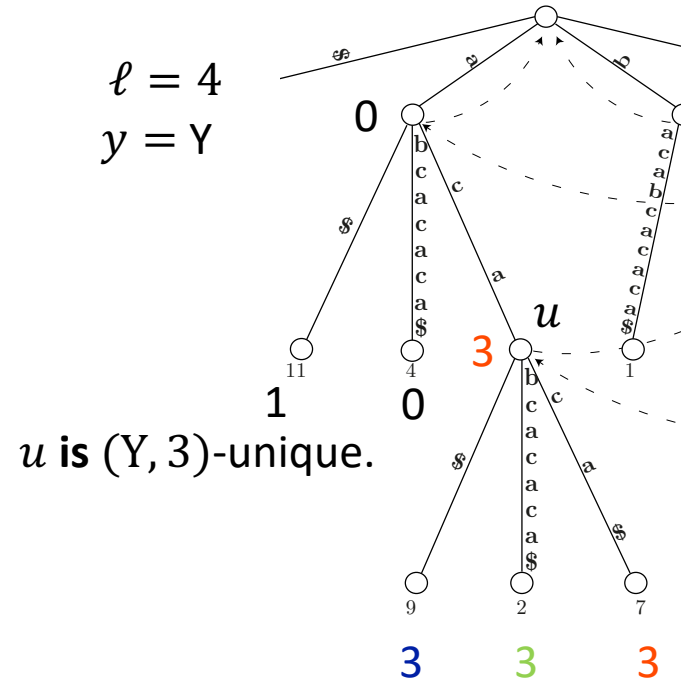
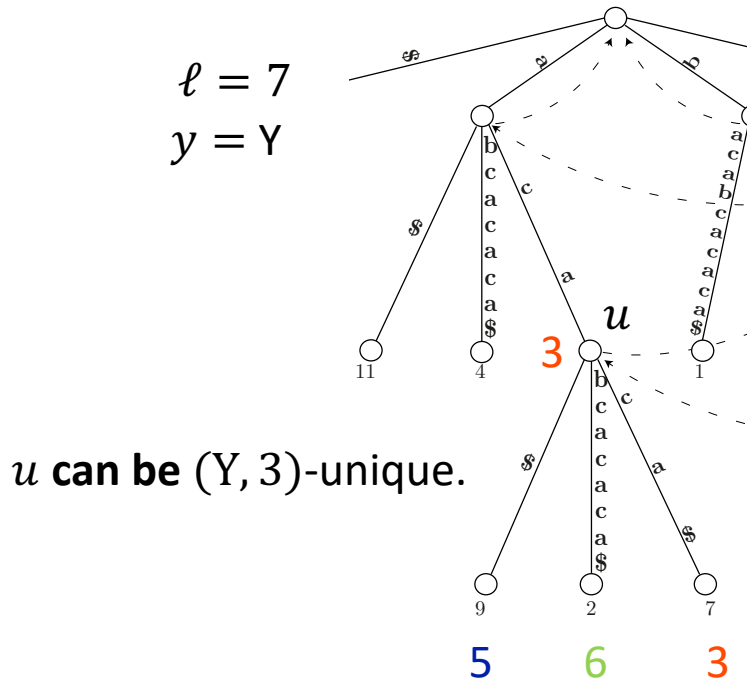
$d = 3$   
 $y = Y$

# Skipping Algorithm

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# Skipping algorithm

Given a node  $u$  and an integer  $\ell$ ,  $h(u, \ell)$  is the largest delay  $d < \ell$  such that the corresponding string can be  $(y, d)$ -unique.



$f^{rev}: Z \ X \ X \ Y \ Z \ Y \ X \ Z \ X \ Y \ X$   
 $S^{rev}: B \ A \ C \ A \ B \ C \ A \ C \ A \ C \ A \ \$$   
 1 2 3 4 5 6 7 8 9 10 11 12

$f^{rev}: Z \ X \ X \ Y \ Z \ Y \ X \ Z \ X \ Y \ X$   
 $S^{rev}: B \ A \ C \ A \ B \ C \ A \ C \ A \ C \ A \ \$$   
 1 2 3 4 5 6 7 8 9 10 11 12

# Skipping algorithm

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- We discover the strings for  $d$  from  $n$  down to 0. (**right minimality**)
- For each node  $u$ , we keep the value  $h(u, d + 1)$  updated.
- We find a node  $u$  such that:
  1.  $u$  has the largest value  $h(u, d + 1)$ ;
  2.  $u$  has priority on its children. (**left minimality**)
- We check if  $u$  is **right minimal**, and if so, we report it.
- We update:
  1. the value of all nodes  $v$  in the subtree rooted on  $u$  to  $h(v, d)$
  2. the value of all ancestors  $v$  of  $u$  to  $h(v, d)$



Maximum-oriented indexed  
priority queue

Runs in  $O(n^2 \log n)$  time.

# Output restrictions

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# Output restrictions

We restrict the output to  $(y, d)$ -unique substrings with at least two occurrences followed by  $y$ .

|       |   |                                                |                                                |                                                  |                                                  |                                                |   |                                                  |                                                 |                                                 |                   |
|-------|---|------------------------------------------------|------------------------------------------------|--------------------------------------------------|--------------------------------------------------|------------------------------------------------|---|--------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------|
| $f$ : | X | Y                                              | X                                              | Z                                                | X                                                | <span style="border: 1px solid blue;">Y</span> | Z | <span style="border: 1px solid orange;">Y</span> | X                                               | X                                               | Z                 |
| $S$ : | A | <span style="border: 1px solid blue;">C</span> | <span style="border: 1px solid blue;">A</span> | <span style="border: 1px solid orange;">C</span> | <span style="border: 1px solid orange;">A</span> | C                                              | B | A                                                | <span style="border: 1px solid green;">C</span> | <span style="border: 1px solid green;">A</span> | B                 |
|       | 1 | 2                                              | 3                                              | 4                                                | 5                                                | 6                                              | 7 | 8                                                | 9                                               | 10                                              | 11                |
|       |   |                                                |                                                |                                                  |                                                  |                                                |   |                                                  |                                                 |                                                 | $d = 3$           |
|       |   |                                                |                                                |                                                  |                                                  |                                                |   |                                                  |                                                 |                                                 | <b>Considered</b> |

|       |   |                                                |                                                |                                                  |                                                  |   |   |   |                                                 |                                                 |                       |
|-------|---|------------------------------------------------|------------------------------------------------|--------------------------------------------------|--------------------------------------------------|---|---|---|-------------------------------------------------|-------------------------------------------------|-----------------------|
| $f$ : | X | Y                                              | X                                              | Z                                                | X                                                | Y | Z | Y | X                                               | <span style="border: 1px solid blue;">X</span>  | Z                     |
| $S$ : | A | <span style="border: 1px solid blue;">C</span> | <span style="border: 1px solid blue;">A</span> | <span style="border: 1px solid orange;">C</span> | <span style="border: 1px solid orange;">A</span> | C | B | A | <span style="border: 1px solid green;">C</span> | <span style="border: 1px solid green;">A</span> | B                     |
|       | 1 | 2                                              | 3                                              | 4                                                | 5                                                | 6 | 7 | 8 | 9                                               | 10                                              | 11                    |
|       |   |                                                |                                                |                                                  |                                                  |   |   |   |                                                 |                                                 | $d = 7$               |
|       |   |                                                |                                                |                                                  |                                                  |   |   |   |                                                 |                                                 | <b>Not considered</b> |

Including this consideration as part of the problem, we can modify the computation of  $h(u, d)$ , when all children of  $u$  are leaves.



# Experimental results

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# Experimental results

## Algorithms:

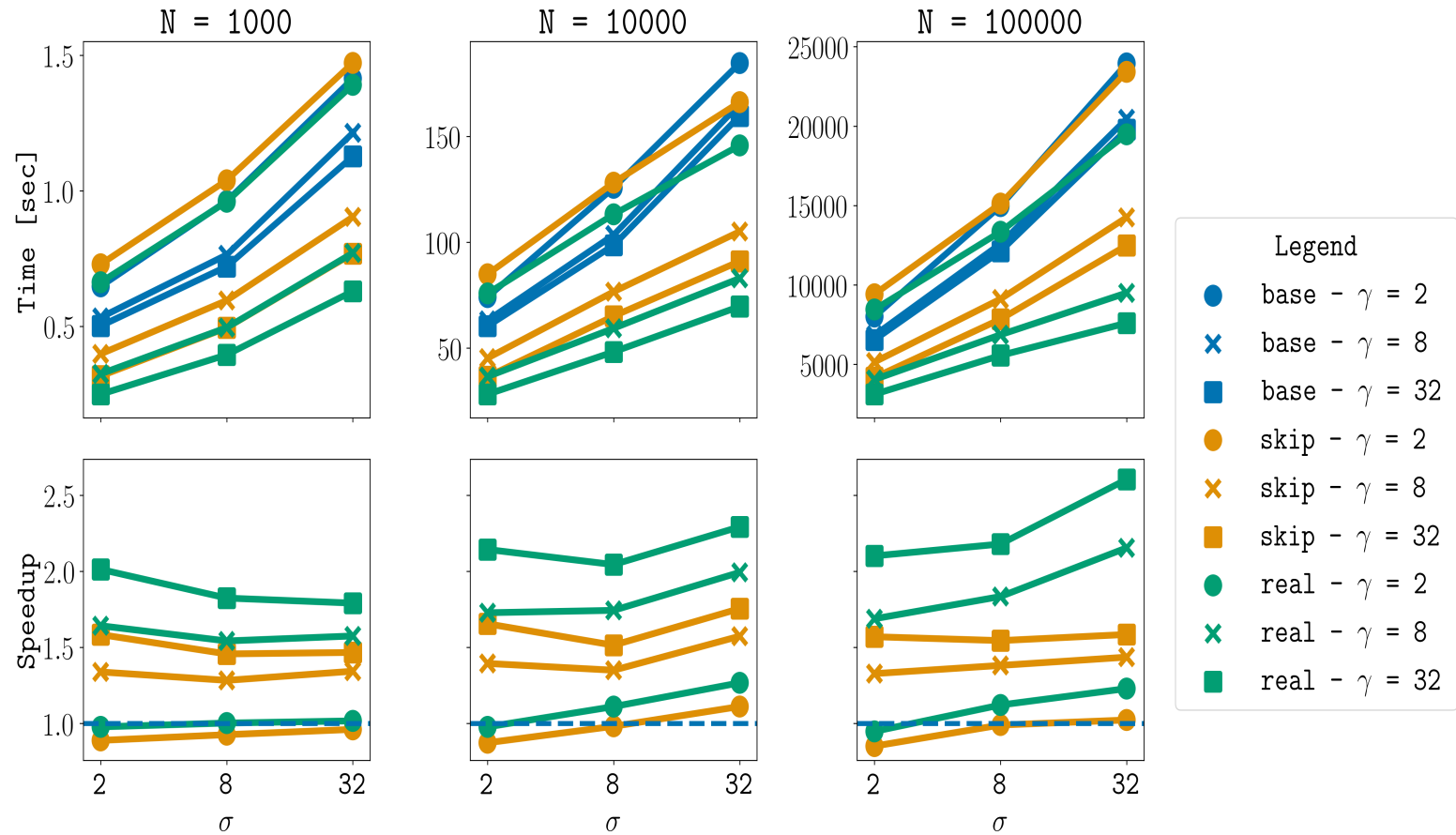
- **Baseline:** Suffix tree based algorithm.
- **Skipping:** Skipping algorithm.
- **Real:** Skipping with output restrictions as part of the problem.

## Data:

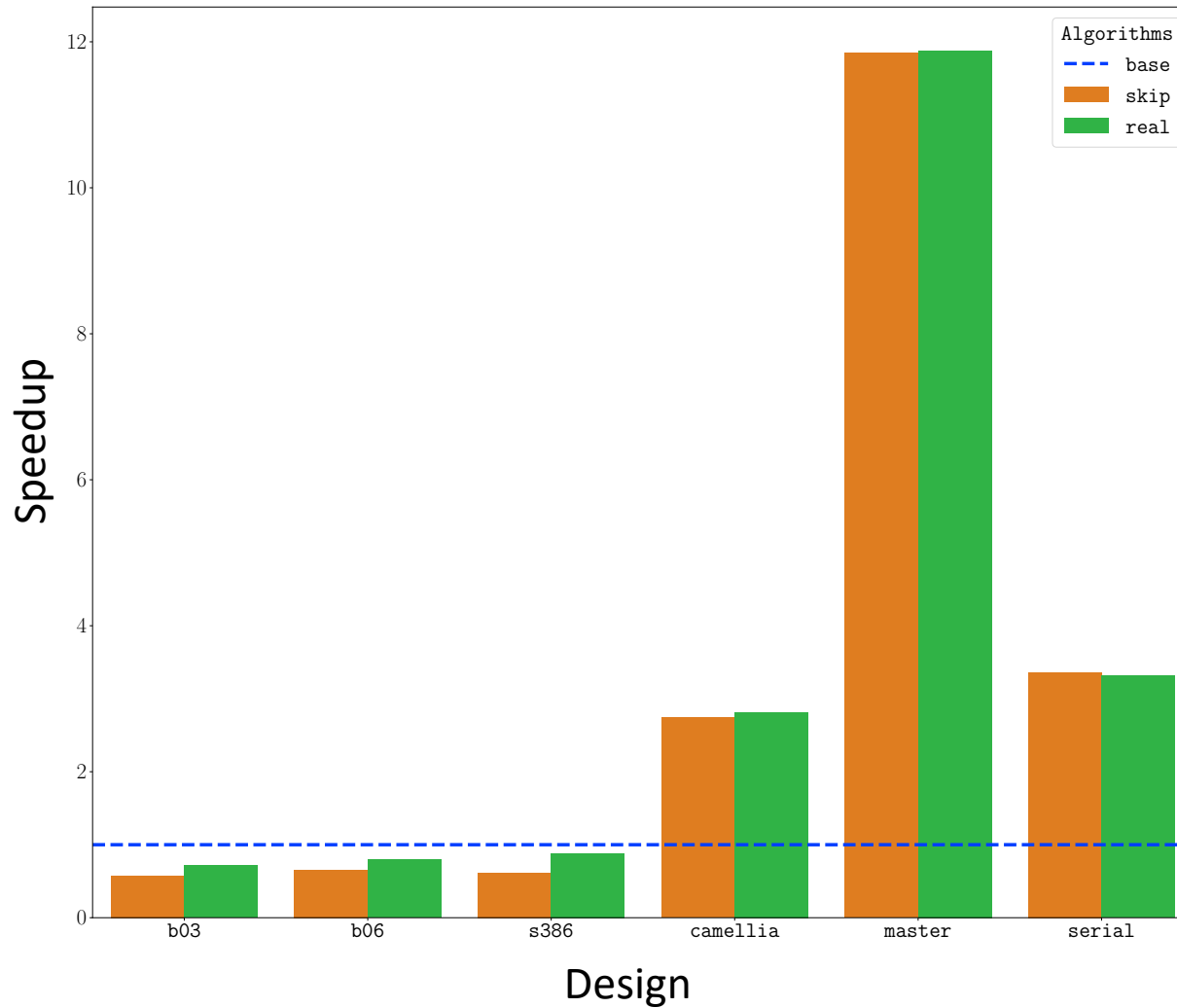
1. **Synthetic data:** Randomly generated data varied:
  - string length  $n = 100, 1000, 10000, 100000$ ,
  - alphabet size = **2, 4, 8, 16, 32**,
  - number of colors = **2, 4, 8, 16, 32**.
2. **Real data:** Simulation on a set of established benchmarks in embedded systems verification.

| Design  | Description                | PIs | POs | $n$    | $\sigma$ | $\gamma$ | $n_\gamma$ |
|---------|----------------------------|-----|-----|--------|----------|----------|------------|
| b03     | Resource arbiter           | 6   | 4   | 100000 | 17       | 5        | 3210       |
| b06     | Interrupt handler          | 4   | 6   | 100000 | 5        | 4        | 44259      |
| s386    | Synthesized controller     | 9   | 7   | 100000 | 129      | 2        | 8290       |
| camelia | Symmetric key block cypher | 262 | 131 | 103615 | 70       | 224      | 2292       |
| serial  | Serial data transmitter    | 11  | 2   | 100000 | 118      | 2        | 16353      |
| master  | Wishbone bus master        | 134 | 135 | 100000 | 417      | 80       | 759        |

# Synthetic data



# Real data



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Thank you for your attention!